Experiment and Analysis of Active Vibration Suppression via an Absorber with a Tunable Delay

Yixia Sun * and Jian Xu **

*School of Mechanical Engineering, Shanghai University of Engineering Science, Shanghai, China
**School of Aerospace Engineering and Applied Mechanics, Tongji University, Shanghai, China

Summary. A time-delayed absorber is presented to suppress the vibration of a primary system when the passive absorber loses efficiency. Both the inherent and the intentional time delays are considered in the feedback loop. The effects of the feedback gains and intentional time delay on vibration suppression are studied experimentally and theoretically. The experimental results show that reasonable feedback gain and intentional time delay may improve the absorber’s vibration suppression effectiveness.

Introduction

Time-delayed absorber is a novel vibration control technique. The key idea of a time-delayed absorber is the introduction of an actuator controlled via state feedback with the adjustable time delay and feedback gain. Olgac [1] designed a delayed resonator (DR) to suppress the vibration of a harmonically forced system. With proper choices of feedback gain and time delay, DR rests the controlled system at the desired frequency. Tootoonchi [2] used DR to minimize the relative vibration between the cutting tool and work piece. Zhao [3] investigated the effect of positive and negative feedback on the performance of a nonlinear time-delayed absorber. El-Sayed [4] observed the effect of a time-delayed absorber on suppressing the vibration due to rotor blade flapping motion. Although, there have been extensive theoretical studies about the time-delayed absorber, few efforts have been devoted to experiments.

The vibration control effect of the time-delayed absorber with inherent time delay has been studied experimentally in our previous work [5]. It is found that the proper utilization of inherent time delay greatly improve the performance of vibration suppression. Motivated by this finding, an intentional time delay is introduced in the controller, the combined effects of inherent and intentional time delays are studied experimentally in this paper.

Modeling and stability analysis

Fig. 1 illustrates the mechanical model of the 2-dof combined system. The system consists of a time-delayed absorber and a primary system. \( m_1 \) and \( m_2 \) represent the mass of the absorber and the primary system, respectively. \( x_1 \) and \( x_2 \) denote the displacement of the absorber mass and the primary mass. An actuator provides the feedback control force. Assuming that a harmonic excitation is applied to the primary system, the governing equations of the combined system are given by

\[
m_1 \ddot{x}_1 + k_1 (x_1 - x_2) + c_1 (\dot{x}_1 - \dot{x}_2) - k_x x_1 = 0
\] (1)

\[
m_2 \ddot{x}_2 + k_2 x_2 + c_2 \dot{x}_2 + k_1 (x_2 - x_1) + c_1 (\dot{x}_2 - \dot{x}_1) + k_x x_2 = f \sin(\omega t)
\] (2)

where \( k_x \) is the time-delayed feedback control force and \( x_i \) is the control signal with the following form

\[
x_i = \alpha_i [x_i (t - \tau_1) - x_i (t - \tau_2)] + \alpha_2 [x_i (t - \tau_1 - \tau_2) - x_i (t - \tau_2)]
\] (3)

where \( \tau_1 \) and \( \tau_2 \) represent the inherent and the intentional time delay in the feedback control loop, respectively. \( \alpha_i \) and \( \alpha_2 \) indicate the amplification factors of the control signal. Substituting Eq. (3) into Eqs. (1) and (2) yields

\[
m_1 \ddot{x}_1 + k_1 (x_1 - x_2) + c_1 (\dot{x}_1 - \dot{x}_2) - g_1 x_1 (t - \tau_1) + g_2 x_1 (t - \tau_1 - \tau_2) + g_3 x_1 (t - \tau_2) = 0
\] (4)

\[
m_2 \ddot{x}_2 + k_2 x_2 + c_2 \dot{x}_2 + k_1 (x_2 - x_1) + c_1 (\dot{x}_2 - \dot{x}_1) - g_1 x_2 (t - \tau_1) + g_2 x_2 (t - \tau_1 - \tau_2) + g_3 x_2 (t - \tau_2) = f \sin(\omega t)
\] (5)

where \( g_1 = \alpha_1 k_1 \) and \( g_2 = \alpha_2 k_1 \) are feedback gains. The solutions of Eqs. (4) and (5) can be expressed as

\[
x_i = a_i \sin(\omega t) + b_i \cos(\omega t), \quad x'_i = a_i \sin(\omega t) + b_i \cos(\omega t)
\] (6)

By substituting Eq. (6) into Eqs. (4)-(5) and extracting the coefficients of \( \sin(\omega t) \) and \( \cos(\omega t) \) from both sides of the equations, we obtain the linear algebraic equations about \( a_i, b_1, a_2 \) and \( b_2 : [a_1, b_1, a_2, b_2]^T = [A]^{-1} [F] \) (7)

Hereinafter, we define \( H_1 \) and \( H_2 \) as the acceleration transfer functions of the absorber and the primary system:

\[
H_i = \frac{\ddot{x}_i}{f} = \frac{\omega^2 a_i^2 + b_i^2}{f}, \quad (i = 1, 2)
\] (8)

Through the above derivation, the period responses of the combined system are obtained. It is known that feedback gain and time delays have a great influence on the stability of the system responses. Hence, it is necessary to analysis the stability of the system responses. The characteristic equation of the combined system is
\[ P_0(s) + P_1(s)e^{-\tau_1 s} + P_2(s)e^{-(\alpha_2 + \tau_1) s} = 0 \]  

The combined system is stable if and only if all characteristic roots of Eq. (9) have negative real parts.

**Experimental results**

The schematic view of the time-delayed feedback control experiment is illustrated in Fig. 2. A shaker (5) and a power amplifier act as the excitation source and provide the sinusoidal excitation force to the primary mass (2). The frequency of the excitation force is set in M+p vib-pilot, which works as a signal generator. Force sensor (6) and acceleration sensors (7), (8) are respectively used to monitor the excitation force and the responses of absorber mass (1) and primary mass. The feedback control loop is described as follows. Firstly, the acceleration signals of sensors (7) and (8) enter into signal conditioning instrument, in which the acceleration signals are amplified and filtered to improve signal-to-noise ratio. Secondly, the processed signals go into trio motion controller via voltage lifting device. The time-delayed feedback control command (i.e. Eq. (3)) is written in trio motion controller. Thirdly, the control command is transferred into servo controller which guides the rotation of the shaft of servo motor (3). Finally, the lower end of controlled steel sheet (4) moves horizontally under the drive of servo motor shaft and applies the feedback control force.

The physical parameters of the combined system are: \( m_1 = 0.667 \text{kg} \), \( m_2 = 6.8 \text{kg} \), \( k_1 = 2431.72 \text{N/m} \), \( k_2 = 13502.4 \text{N/m} \), \( k_5 = 198.96 \text{N/m} \), \( c_1 = 2.9 \text{Ns/m} \), \( c_2 = 4.1 \text{Ns/m} \) and \( \tau_1 = 63 \text{ms} \). Hereinafter, \( \Omega = \omega/2\pi \) (in hertz) denotes excitation frequency.

Fig. 3 shows the variations of \( H_2 \) versus \( \alpha_1 \) for \( \Omega = 10.25 \text{Hz} \), \( \alpha_2 = 0.4 \) and \( \tau_2 = 30 \text{ms} \). The line represents the analytical results determined according to Eq. (8). The red dots represent experimental results. It’s obvious that the time-delayed absorber minimizes the vibration of the primary system for \( \alpha_1 = 0.7 \). To observe the effect of the intentional time delay on the vibration suppression effect, Fig. 4 shows how \( H_2 \) change as a function of \( \tau_2 \) when \( \Omega = 10.25 \text{Hz} \), \( \alpha_1 = 0.8 \) and \( \alpha_2 = 0.5 \). It is found that the value of \( H_2 \) fluctuates with the increase of \( \tau_2 \). There is a peak value when \( \tau_2 \approx 40 \text{ms} \) and a valley value when \( \tau_2 \approx 85 \text{ms} \). Fig. 5 shows the measured time histories of the excitation force and the corresponding system responses for \( \Omega = 10.25 \text{Hz} \). It is seen that the value of \( H_2 \) decreases 71.93% when the time-delayed feedback is applied.

**Conclusions**

The effectiveness of a time-delayed absorber in suppressing the vibration of a harmonically excited primary system is demonstrated. An experimental device is designed to implement the time-delayed feedback control. Experimental and theoretical results point out that the time-delayed absorber with proper choices of control parameters is effective for vibration suppression. It is noticed that for fixed feedback gains, sharp vibration suppression may occur at some values of intentional time delay. However, the time-delayed absorber fails at other values of intentional time delay.

**References**