Internal resonances in tiny structures: new results and practical applications

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Summary. Internal resonances occur in vibratory systems when one or more eigenmodes have commensurate natural frequencies, a condition that can lead to complicated dynamic interactions between the resonant modes. These resonances have a long history of study but were generally viewed by practicing engineers as academic curiosities to be avoided. However, recent experimental investigations of micro- and nano-scale resonators have given new life to this class of systems and opened up the possibility of practical uses for sensing, frequency synthesis, and frequency conversion. In this presentation I will provide a survey of these recent results and describe how experimental work and theory have informed one another and led to advances in our fundamental understanding of internal resonances and our ability to design these systems for practical use.

Background

Internal resonances (IRs) are known to occur in a variety of systems and have been widely studied [1]. In the 1980's these systems received a lot of attention, focusing on their occurrence in mechanical structures and their response to harmonic excitation [1]. Much of that work was inspired by the development and availability of computational tools that allowed researchers to more thoroughly investigate the system response. The recent availability of small-scale structures, specifically, micro- and nano-electro-mechanical-system (M/NEMS) resonators, has led to a renewed interest in IR, since these devices are highly tunable and offer access to previously unattainable parameter conditions. M/NEMS resonators are very lightly damped and are sensitive to thermal and electronic noises, and therefore must be driven strongly near resonance to enhance their signal-to-noise ratio (SNR), thereby promoting nonlinear behavior [2].

Most of the initial efforts on nonlinear responses in M/NEMS were focused on single-mode Duffing-type responses, but more recently there have been many studies on modal interactions, including IRs. A sampling of experimental studies demonstrating known IR behavior in small resonators includes devices with frequency ratios of 1:1 [3, 4, 5], 1:2 [5, 6, 7], and 1:3 [5, 8, 9, 10]. Some researchers have intentionally introduced nonlinear multi-mode behavior for applications such as filtering [6], frequency conversion [7], and amplification [3]. However, experimentalists have also serendipitously encountered these interactions, and in some cases they have seen that benefits can be gained when operating with IR. Prime examples of this include: (i) the significant reduction in phase noise observed in a flexural-torsional MEMS when operating in closed loop with a 1:3 IR, which has direct implications for improving the performance of frequency sources and other timing applications [8], and (ii) an increase in signal gain in MEMS vibratory gyroscopes resulting from inter-modal parametric pumping [11]. All of these phenomena can be described by generic mathematical models from the theory of normal forms [1]. However, even for two-mode interactions, these models are sufficiently rich to offer some new surprises, and the interplay of experiment and theory have led to some original results about IRs.

Highlights of New Phenomena

Among these phenomena are modal interactions that cause unexpected behaviors during transient operation, specifically during ringdown towards equilibrium. These decay behaviors are distinct from those associated with nonlinear modal damping [12] and include: (i) responses in which the modes experience amplitude-dependent decay rates with a relatively fast transition in character, as shown in Fig. 1a, from [10]; (ii) responses in which the expected decay is interrupted by a significant, but temporary, increase in damping, as shown in Fig. 1b; and (iii) cases where the primary mode does not decay at all for a significant time and then decays in its usual manner, as depicted in Fig. 1c, from [13]. These experimental results, which are more fully described in the caption of Fig. 1, are all consequences of two-mode resonant interactions that can be described by the appropriate normal form under different parameter conditions. Specifically, the nature of the ringdown response depends on the absolute and relative decay rates of the individual modes, the detuning of the IR frequency condition, the strengths of the modal (Duffing) nonlinearities, the strength of the nonlinear modal coupling, and, of course, the initial conditions [14]. For example, the behavior shown in Fig. 1b is is described by an adiabatic limit in which the secondary mode is much faster than the primary mode and is temporarily excited during passage through an IR, acting as an effective nonlinear energy sink that is active only near its resonance [14]. This non-uniform decay is accompanied by strong nonlinear shifts in frequency that have direct and unexpected effects on the nature of the response when the system is subjected to harmonic drive or placed in a feedback loop [14].

Another IR phenomenon recently observed in a MEMS experiment is a SNIC (Saddle Node on an Invariant Circle) bifurcation, in which a saddle node bifurcation encountered near the peak of a Duffing resonance results in a response with bursting behavior with long periodic dwells between bursts. This period scales in a systematic manner with the system parameters near the SNIC bifurcation point and becomes infinite at the bifurcation, leading to a power spectrum with a tunable frequency comb, which has desirable properties for applications [15].

Conclusions

These experimental results are motivating a re-examination of IR normal form models in order to describe the observed phenomena, and the models are providing guidance to M/NEMS experimentalists and designers. In fact, normal forms are being used for design optimization of devices with nonlinear resonances, including IR [16, 17], which will be important for applications. Further progress in this field will also benefit from models that describe how noise affects the dynamics of IR systems, which is especially important for understanding the benefits and limitations of tiny IR structures.

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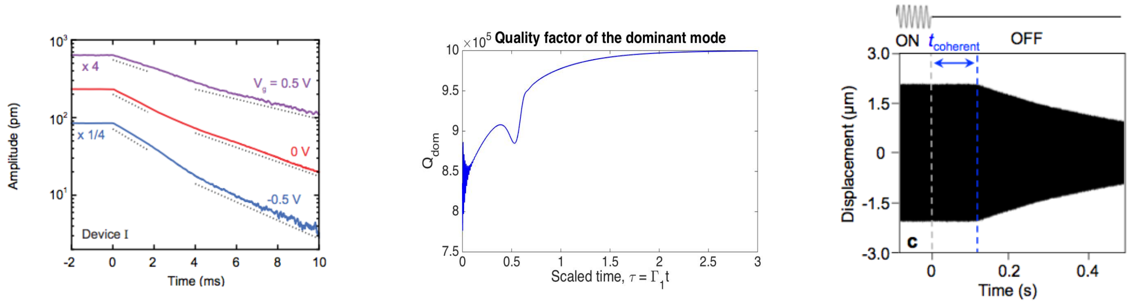

Fig. 1. Ringdown responses from three different devices with IR. (Left) Averaged modal amplitude (over many samples of decay) versus time from a graphene membrane with a 1:3 IR and a primary mode frequency of 46 MHz, for three different gate voltages; from [10]. Note the relatively sharp transition in the decay rate, which is due to passage through an unstable coupled-mode response [14]. (Center) Instantaneous quality factor Q versus time for a 10 MHz Lamé mode resonator, where nonlinear damping of the mode is evident from the increasing Q and the dip corresponds to an interaction that occurs when the frequency of the primary mode, which is varying due to a Duffing nonlinearity, encounters a resonance with a (undetermined) secondary mode; data is unpublished, from T. Kenny, with permission. (Right) Flexural mode displacement versus time for a flexural-torsional MEMS with a 1:3 IR with a primary mode frequency of 61 kHz, showing a coherence time of about 0.1 seconds over which the primary mode does not decay after a harmonic drive is switched off. During the coherence time the flexural mode absorbs energy from the torsional mode until it is exhausted, after which the flexural mode undergoes its usual decay; from [13].

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