Dynamical response identification of a class of nonlinear hysteretic systems

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Summary. The experimental dynamical response of a hysteretic oscillator connected to a shaking table is investigated. The hysteretic forces are provided by an assembly of shape memory and steel wire ropes subject to flexure or coupled states of tension and flexure. The mechanical system is characterized by the energy dissipation due to phase transformations of the shape memory material and inter-wire friction and the stretching-induced geometric nonlinearities giving rise to a rich variety of possible hysteretic cycles. The dynamical behavior of the system subject to stationary and nonstationary base excitations is identified exploiting the measurements of the oscillating mass relative displacement and inertia force that must be balanced, at each time instant, by the resultant restoring forces. A modified version of the Bouc-Wen model is proposed for reproducing the nonsymmetric force-displacement cycles where the proper model parameters are identified using the Differential Evolutionary Algorithm.

Introduction

Wire ropes are structural elements mainly employed to lift huge masses thanks to the high tensile stiffness and strength. Short wire ropes subject to bending cycles provide a distinct hysteretic behavior due to the inter-wire frictional forces. Different classes of force-displacement cycles were obtained combining the inter-wire friction with the phase transformations of wires made of NiTiNOL (Nickel-Titanium Naval Ordnance Laboratory) and geometric nonlinearities induced by the loading conditions [2]. This system provides a variety of dynamical responses [3] which can be suitable for several engineering applications mainly within the field of vibration control [1, 4]. A fundamental task for the exploitation of these mechanical behaviors is represented by a suitable mathematical identification. A convenient classification of the existing identification techniques takes into account two distinct families of parametric and nonparametric methods. Parametric methods require the development of appropriate constitutive models which have the advantage of being naturally suited for numerical simulations [2, 4, 3]. On the other hand, nonparametric methods can enhance the possibility to better describe highly nonlinear, complex, memory-dependent behaviors because they do not constrain the identification to a given mathematical model [6, 7].
The dynamical response of the mentioned hysteretic device [2, 3] is here experimentally investigated and identified. The system is subject to harmonic base excitations and to nonstationary base motions. The force-displacement response is obtained through the measurements of the oscillating mass relative displacement and absolute acceleration. The response of some device configurations to the nonstationary base excitations exhibits nonsymmetric hysteretic behavior. To overcome the limitations of the classical Bouc-Wen model [5], an extended version is here proposed while the parameters identification is performed using the Differential Evolutionary Algorithm.

The investigated dynamical behaviors

The device is constituted by an oscillating mass connected to a rigid frame by means of a principal group of wire ropes. The ropes are straight and orthogonal to the mass oscillation direction in the reference configuration. The rigid frame is fixed to a shaking table. When the mass is excited the wire ropes can be subject to pure bending loads or to coupled states of bending and tension which induce geometric nonlinearities in the restoring force. An additional group of ropes subject to tensile cycles can be introduced in the device. This secondary group can be used to regulate the level of hardening in the total restoring force applied to the mass as well as the damping. A detailed description about the mechanical system and the possible hysteretic behaviors is reported in [2, 3].
Figure 1 shows three hysteretic behaviors which are obtained with different assemblies of wire ropes. The force-displacement cycles shown in part (a) are of the hardening-type while part (b) exhibits initially linear cycles which become hardening past a threshold displacement. Figure 1 (c) shows softening force-displacement cycles.

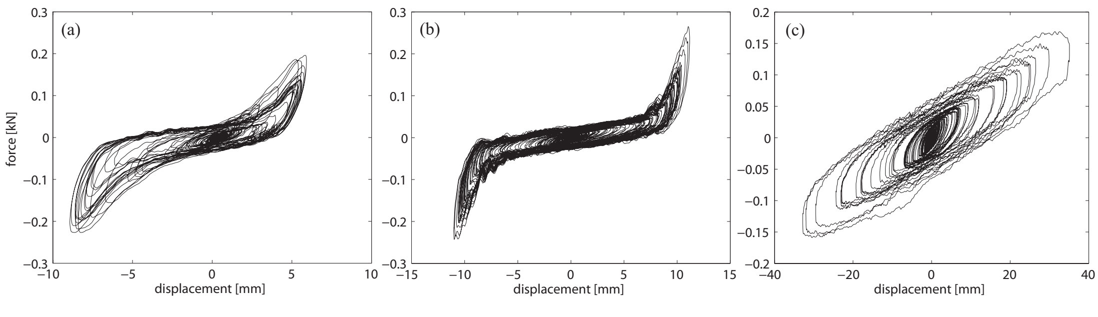

Figure 1: Force-displacement cycles of three different device configurations subject to seismic base excitations: (a) hardening behavior, (b) linear behavior followed by hardening, (c) softening behavior.

The phenomenological identification

The original Bouc-Wen model of hysteresis in its original form is capable of describing the behavior in Fig. 1 (c) but not the cycles reported in Fig. 1(a)-(b). A modified version of the model is proposed whose equations are not reported for sake of brevity. The proposed model can effectively describe nonsymmetric behaviors in terms of hardening or hysteretic dissipation.
The equation of motion of the device subject to base motion can be written as

$$m\ddot{x} + f(x, \dot{x}) = -m\ddot{x}_g, \tag{1}$$

where m is the oscillating mass ($m = 6.46$ kg), the overdot denotes differentiation with respect to time t, $\ddot{x}$ is the relative acceleration, and $\ddot{x}_g$ is the base acceleration. Denoting by $x_a = x_g + x$ the absolute mass displacement, given by the sum of the shaking table displacement and the mass displacement relative to the shaking table, the total restoring force $f(x, \dot{x})$ must be, at each instant of time, equal and opposite to the inertia force $f_i(t) = m\ddot{x}_a$ with $\ddot{x}_a = \ddot{x} + \ddot{x}_g$ indicating the absolute mass acceleration.
The identification procedure consists in the identification of the restoring force $f(x, \dot{x})$ minimizing the residual of Eq. (1) according to the balance of linear momentum

$$f_i(t) + f(x, \dot{x}) = 0 \quad \forall t \in [0, T], \tag{2}$$

where $[0, T]$ indicates the time interval of the dynamical test.
The objective function whose minimum is sought is represented by the residual expressed in terms of mean square value (MSE)

$$r(\mathbf{p}) = \frac{100}{N\sigma_{f_i}} \sum_{j=0}^{N} (f_i(t_j) + f(x(t_j), \dot{x}(t_j), \mathbf{p}))^2, \tag{3}$$

where $\mathbf{p}$ is the vector collecting the model parameters, N is the number of time instants, t_j indicates the time instants into which the experimental data $x(t_j)$ and $\ddot{x}_a(t_j)$ are acquired (i.e. $t_0 = 0$ and $t_N = T$), and σ_{f_i} indicates the variance of the recorded inertia force signal. The relative displacement x and velocity $\dot{x}$ are employed for reproducing numerically the restoring force $f(x, \dot{x})$ and the absolute acceleration for defining the objective function $-f_i = m\ddot{x}_a$. The identification is performed using the Differential Evolutionary Algorithm.
Figure 2 shows some identified results where the black and gray solid lines represent the experimental and the predicted results, respectively. The mean square error of the identification is lower than 1%.

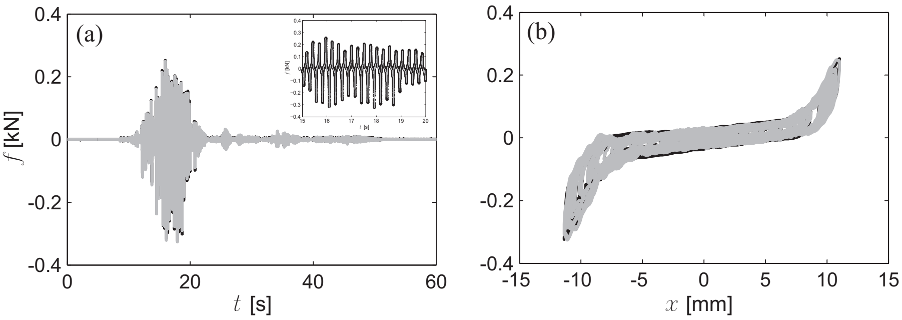

Figure 2: The experimental (black solid lines) and identified (gray solid lines) restoring forces vs. time in (a) and vs. mass displacement in (b).

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