An Online Control Strategy for Time Delayed Vibration Absorber

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Summary. In this paper, the mechanical model of a time delayed vibration absorber is established. The periodic response of the delay coupled system is obtained. The zero-equilibrium stability is analysed to get available control parameters. The region where the response of the delayed absorber is 50% less than that of the uncontrolled absorber is plotted. The optimal control parameters are obtained. An online control strategy is proposed. The result shows that the control strategy is valid in a certain range of excitation frequency.

Introduction

Vibration absorbers have been effectively used to remove undesirable oscillations. However a traditional vibration absorber is only valid at a certain frequency. To solve this drawback, active control strategies of vibration absorbers have been widely studied. Among them delayed absorbers have attracted extensive attention since Olgac et. al. [1] firstly introduce time delayed feedback control to suppress the vibration of a finite DOF system. Sun & Xu [2] design a time-delayed feedback controlled mechanical device. Experimental study shows that the anti-resonant frequency is controllable. This paper presents an online control strategy of delayed absorber which is capable to suppress the vibration of the primary system when the excitation frequency changes.

Mechanical model and stability analysis

In this paper, we consider the following 2 DOF system of a vibration absorber with delayed feedback control [3]:

$$M\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + c_2\left(\dot{x}_1 - \dot{x}_a\right) + k_2\left(x_1 - x_a\right) = F\sin\left(\omega t\right), \tag{1}$$

$$m_a\ddot{x}_a + c_3\dot{x}_a + k_3x_a + c_2\left(\dot{x}_a - \dot{x}_1\right) + k_2\left(x_a - x_1\right) + gx_a\left(t-\tau\right) = 0, \tag{2}$$

where m_1, k_1, c_1 and x_1 denote, respectively, mass, stiffness, damping and displacement of the primary system; m_2, k_2, c_2 and x_a are those parameters of the absorber; and k_3 and c_3 are caused by the attachment of the controller. $F\sin\left(\omega t\right)$ is the excitation force applied on the primary system. $gx_a\left(t-\tau\right)$ is the time-delayed feedback control force, g is the control gain, τ the time delay.

Periodic response

The solutions of Eqs. (1) and (2) can be expressed as

$$\begin{Bmatrix} \bar{x}_1 \\ \bar{x}_a \end{Bmatrix} = \frac{F}{\Delta}\begin{Bmatrix} -m_a\omega^2 + i\omega\left(c_2+c_3\right) + k_2 + k_3 + g\exp\left(-i\omega\tau\right) \\ -i\omega c_2 - k_2 \end{Bmatrix}, \tag{3}$$

where $\Delta = \left(-M\omega^2 + i\omega\left(c_1+c_2\right) + k_1 + k_2\right)\left(-m_a\omega^2 + i\omega\left(c_2+c_3\right) + k_2 + k_3 + g\exp\left(-i\omega\tau\right)\right) - \left(i\omega c_2 + k_2\right)^2$.

Stability analysis

The characteristic equation of the delay coupled system is

$$\left(Ms^2 + s\left(c_1+c_2\right) + k_1 + k_2\right)\left(m_as^2 + s\left(c_2+c_3\right) + k_2 + k_3 + g\exp\left(-s\tau\right)\right) - \left(sc_2 + k_2\right)^2 = 0. \tag{4}$$

The system is stable if and only if all the roots of Eq. (4) have negative real parts. Substituting $s = i\Omega$ into Eq. (4), separating the real and the imaginary parts and considering $\sin^2 + \cos^2 = 1$ gives the critical values of control parameters.

Optimal control parameters

According to Eq. (3), the vibration amplitude of the primary system is zero when

$$-m_a\omega^2 + i\omega\left(c_2+c_3\right) + k_2 + k_3 + g\exp\left(-i\omega\tau\right) = 0. \tag{5}$$

The optimal control parameters can be expressed as

$$g = \pm\sqrt{\left(k_2 + k_3 - m_a\omega^2\right)^2 + \left(c_2+c_3\right)^2\omega^2}, \tau = \frac{1}{\omega}\left(\arctan\left(\frac{\left(c_2+c_3\right)\omega}{m_a\omega^2 - k_2 - k_3}\right) + 2n\pi\right), n = 1,2,3,\cdots. \tag{6}$$

Online control strategy

This paper proposes a simple online control strategy to suppress the vibration of the primary system when the excitation frequency changes. As Fig. 1 shows, this strategy has three functional parts, namely, stablization,

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identification and control parts. The transient motion of the system vanishes after the stabilization parts. The identification parts are designed to identify the changing frequency of the excitation. In these parts, the motion of the system can be regarded as harmonic vibration so that the frequency can be easily identified. Then the control parameters are chosen as the optimal ones according to Eq. (6) until the next control part begins. This control strategy can deal with problems in which the excition frequency changes slowly.

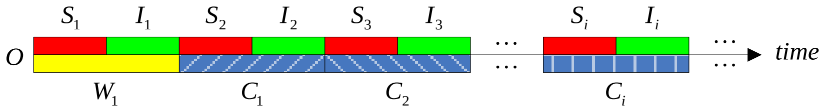

Figure 1 Diagram of an online control strategy (*S*: stabilization parts; *I*: identification parts; *C*: control parts; *W*: waiting time)

Main results

Fig. 2 shows the region where the vibration amplitude of the primary mass under delayed control is 50% less than that of a passive vibration absorber. Fig. 2a shows the 3D plot of the region and Fig. 2b shows the region in $g-\tau$ plane when ω is 10Hz.

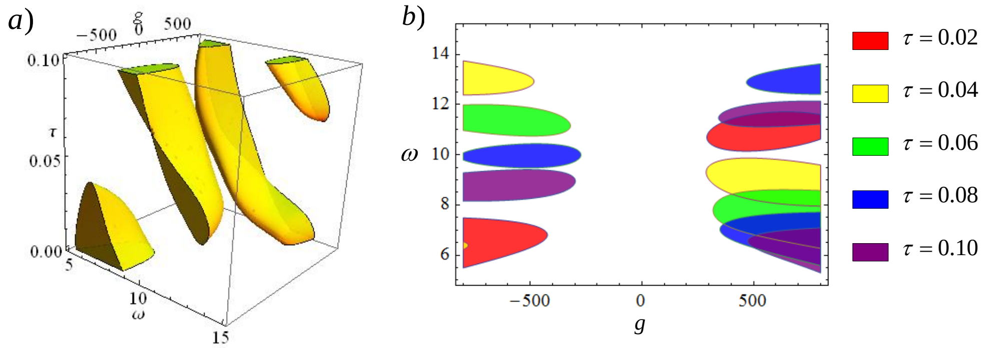

Figure 2 Regions where delayed control performs 50% better than passive absorber a) 3D plot of the region; b) 2D plots in $g-\omega$ plane with different time delays.

In Fig. 3, the numerical simulation result indicates that this online control strategy works efficiently. The excitation frequency jumps from 10.1 Hz to 11.0 Hz at 3.4s.

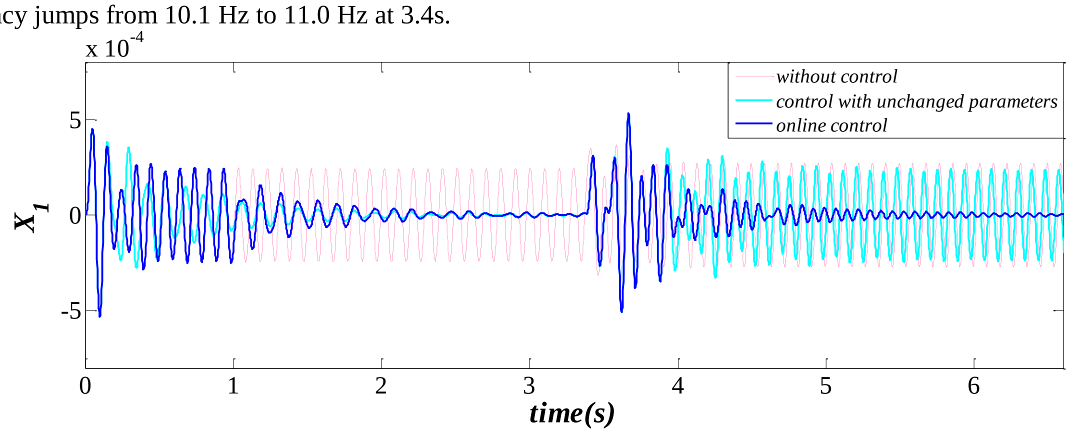

Figure 3 The performance of the online control strategy

Discussions

Although the control strategy proposed in this paper works efficiently, it is valid only in a certain range of excitation frequency. A broad band control strategy deserves further research. In the mean time, other types of delayed feedback control should be studied to enhance the performance of delayed absorbers.

References

[1] Olgac, N., & Holm-Hansen, B. T. (1994). A novel active vibration absorption technique: delayed resonator. *J. Sound Vib.*, 176(1), 93-104.
[2] Sun, Y., & Xu, J. (2015). Experiments and analysis for a controlled mechanical absorber considering delay effect. *J. Sound Vib.*, 339, 25-37.
[3] Zhang, X., Xu, J. & Huang Y.(2015). Experiment on parameter identification of a time delayed vibration absorber. *IFAC-PapersOnLine*, 48(12):57-62.