An anisometric dynamical integrity measure and its seamless variation with respect to other measures

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Summary. This work undertakes a parametric variation of the twin-well Duffing oscillator in the framework of global dynamical analysis. A smooth transition between multiple configurations of the system is obtained by considering a small step between different excitations, basins of attraction are constructed accordingly. The seamless evolution of key dynamical integrity measures highlights the erosion of the safe domain with respect to an increasing force amplitude. Furthermore, this work introduces the *anisometric local integrity measure* (ALIM) as improvement and generalization of the existing *local integrity measure* (LIM). The institution of a non-equidistant integrity measure makes possible to account for inhomogeneous sensitivities to perturbations of the state space variables and permits a more confident and targeted definition of safe regions.

Introduction

In applied mechanics and structural dynamics, the local stability analysis has been proven insufficient to ensure safe values for the system design. Indeed, the safety of the equilibrium state is directly correlated to the effects of practical, unavoidable perturbations, occurring during the work of the structure and, as a consequence of that, the practical (in addition to the theoretical) stability has to be considered [1].
Thus, the robustness of an equilibrium configuration must be verified in the ensemble of all the possible, practically interesting (not infinitesimal) changes in the initial conditions. By considering a global approach, the system safety as well as its evolution under the effect of parametric variation can be properly investigated by building basins of attraction. The importance of basins of attraction is testified by their massive use in, and not limited to, several engineering fields. The global behaviour of both discrete and continuous, micro and macro, smooth and non-smooth dynamical systems is commonly investigated by making use of bidimensional basins, fully describing the state space or being only cross sections of a multidimensional domain.
The magnitude and compactness analysis of the basin of interest turns out as an extremely powerful methodology to predict the actual working domain of the system and, in order to have a quantitative representation, the so-called integrity measures are utilized [2].
Concisely, four key integrity measures have been introduced in literature [3, 4]: i) the global integrity measure (GIM) ii) the local integrity measure (LIM) iii) the impulsive integrity measure (IIM) iv) the integrity factor (IF). In particular, by considering a basin portrait of a two-degree-of-freedom system, the GIM, being the total area of a basin, including the fractal parts, is hardly a confident measure; the IIM is the distance from the safe attractor to the basin's boundaries for a fixed generalized coordinate (usually it refers to the velocity). Finally, both the IF and LIM are defined as the radius of the largest circle inscribed within the safe basin, but the LIM, with the constraint of being centred on the safe attractor, is more conservative: the magnitude estimation of these two measures follows the sorting IF≥LIM. As it can be perceived all the aforementioned measures do not account for inhomogeneous sensitivities of the state space variables but in practical application, a system could be more susceptible to a specific perturbation, e.g. in velocity rather than position. The local integrity measure is here generalized with the introduction of the anisometric local integrity measure (ALIM) that is non-equidistant in the state-space coordinates. It is defined as the maximum of the two semi-major axes of an ellipse centred in the safe attractor and totally contained in the safe basin. The ALIM requires the definition of a ratio between the major and minor axes of the ellipse, here called β. If $\beta = 1$ the ALIM reduces to the LIM while $\beta = 0, \infty$ describes a measure accounting for only one of the two (in 2D basins) generalized coordinates, e.g IIM for $\beta = \infty$.

Dynamic model

Based on previous considerations, in this work we analyse the global dynamics of an archetypical system, commonly reported in the nonlinear dynamics literature as two-well/double-well/bistable Duffing oscillator [5]:

$$\begin{cases} \dot{q}_1 = q_2 \\ \dot{q}_2 = -2\zeta q_2 + q_1 - \gamma q_1^3 + f\cos(\Omega t)\,. \end{cases} \tag{1}$$

The system is framed with a linear viscous damping $\zeta = 0.025$ whereas other constant parameters are $\gamma = 1, \Omega = 1.2$. The state space $\{q_1, q_2\}$, describing all the possible pairs { position, velocity} for the oscillator, is here limited by $q_{1,2} \in [-2, 2]$ and a square grid composed of 500^2 cells is considered for its discretization. A parametric variation, with respect to the excitation amplitude ($f \in [0.02, 0.135]$), is performed with a small step $\Delta f = 0.000225$; this step is so small that practically we are performing a "continuous" analysis of the system for varying f. The integrity of the basins portrait is evaluated using three different measures, namely the LIM, the ALIM and the IF.

Results and discussions

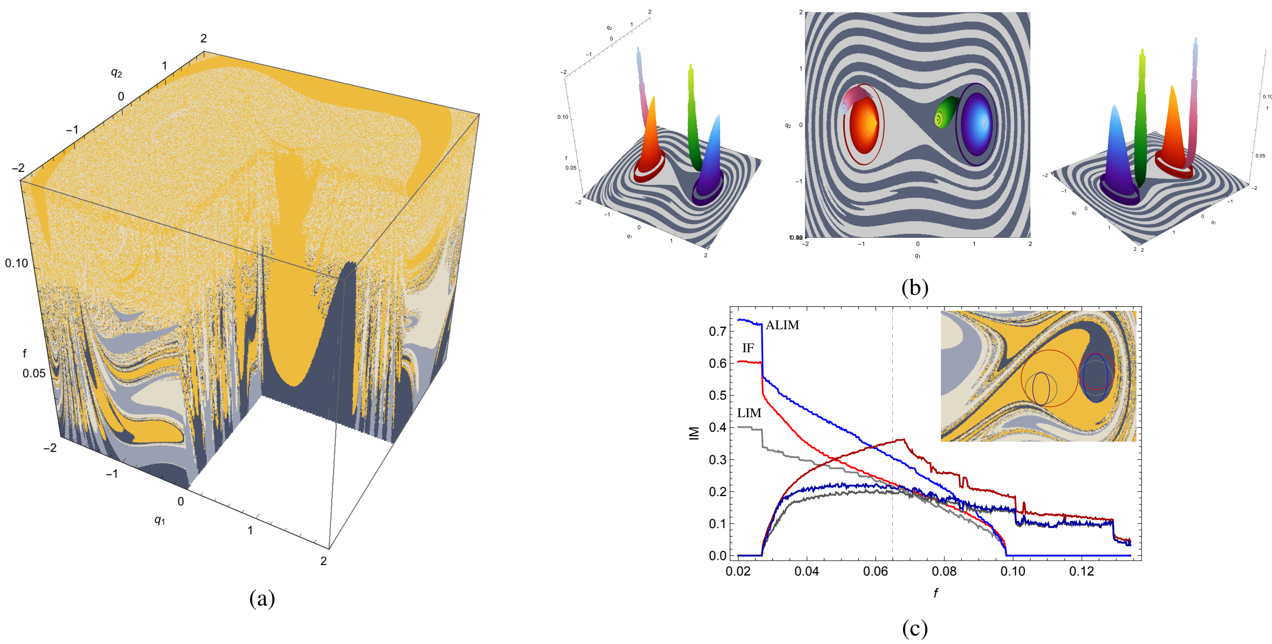

Figure 1: a) Sliced 3D view of the domain of parametric variation of basins of attraction. b) 3D variation of the ALIM for both left and right attractors. c) Integrity profile curves for the right attractors. The ratio in the ALIM is $\beta = 2$. The inset picture shows the shape of the integrity measures for $f = 0.065$.

In Figure $1(a)$ is reported a sliced 3D view of the domain of parametric variation of basins of attraction of Equation (1). It is built as a "continuous" sequence of horizontal 2D subsections (at $f = const.$) that are formally basins of attraction. The pictures help in the understanding of how basins erosion evolves with a varying parameter: the action of an increasing excitation amplitude brings the basin towards a corrupted (fractal) configuration and thus to be dynamically non-integer. Figure $1(b)$ shows the metamorphoses of the ALIM for the four considered attractors. It can be observed the sudden reduction of the ALIM values for the non-resonant attractors, reported in red and blue blended colors, in connection with the appearance of the resonant attractors (green and purple colors). Furthermore, view from above in the middle of Figure $1(b)$ helps to visualize the evolution of the attractors position in the phase plane up to the disappearance of all of them. In Figure $1(c)$ are reported the integrity profile curves for the right attractors of the considered system. The curves highlight the erosion profile varying the excitation amplitude for three integrity measures: IF (red), ALIM (blu), LIM (gray). A decreasing/increasing trend is reflected in a reduction/increment of structural stability for the system. Two different sets of curves are reported, corresponding to the right non-resonant (small amplitude) and resonant (large amplitude) attractors.
The inset plot in Figure $1(c)$ illustrates that the ALIM can be both greater, equal or less than IF, differently from the LIM (always less or equal to the IF). Indeed the reported basins for $f = 0.065$ present a value for the small/large amplitude attractor with ALIM greater/less than the IF. Thus in the presented configuration with $\beta = 2$, for the large amplitude attractor the non-equidistant measure results more conservative, while, in the non-resonant basin, due to the strongly vertically-stretched shape, the ALIM fits more precisely to the basin resulting in a larger measure.

Conclusions

A nearly continuous parametric variation of the classical Duffing oscillator has been undertaken by means of basins of attraction in order to unveil the seamless evolution of the integrity measures with respect to the external excitation amplitude. A generalization of the local integrity measure, namely the anisometric local integrity measure, has been introduced in order to account for different sensitivity of the generalized coordinates to parameter variations.

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