Predictive control of robot manipulators with flexible joints

Bálint Bodor*, László Bencsik**
*Department of Applied Mechanics, Budapest University of Technology and Economics, Budapest, Hungary
**MTA-BME Research Group on Dynamics of Machines and Vehicles, Budapest, Hungary

Summary. In the industry the use of lightweight machines is increasing. Beside the usefulness and speed of these robots the elasticity appears. The elasticity which can be modelled as additional passive joint(s) makes the device underactuated. The classical robot control techniques like the computed torque control can be generalized to underactuated systems. With this generalization the curiosity of the underactuated system the internal dynamics will be hidden. Here a predictive control method will be proposed which considers the internal dynamics as well in the control design. It will be showed that with the predictive control technique the trajectory tracking of underactuated systems is possible even in cases when the system has unstable internal dynamics.

Problem statement

The robot manipulator which contains flexible links or joints can be modelled typically as an underactuated system. The tracking control of these type of systems is an actively studied topic. While in case of a conventional robotic application the relation between all of the states and all of the desired outputs can be defined always, in case of an underactuated robot beside the driven dynamics the non-controlled dynamics appears also. It is called as internal dynamics [2]. If the internal dynamics is not stable the stability of the controlled system can not be achieved. In the field of underactuated systems it is quite common to use the generalization of the computed torque control techniques, like the servo-constraint based control approach [1], [3]. These methods consider only the driven dynamics in the computing of the required control force and the internal dynamics does not appear in the control description. A major obstacle in applying this method for the trajectory tracking of an underactuated system is that there is no one-to-one relationship between the input force and the desired accelerations. These accelerations are also affected by the internal dynamics of the system and cannot simply be calculated based on the current state. That is the reason why the original servo-constraints have to be modified in many cases [1] in order to guarantee the stability of the internal dynamics.
In this work a different approach will be presented. We propose a predictive method which tries to determine the solution of the non-driven dynamics for the unactuated states and using this a priori information tries to fulfil the servo-constraints with as little violation as possible.
In order to develop this predictive method the equation of motion is considered in the following general form:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{c}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{H}(\mathbf{q})\boldsymbol{\tau}, \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{q}$ is the array of generalized coordinates, $\mathbf{c}$ is the array of nonlinear inertial forces lumped together with gravity, $\mathbf{H}$ is the input matrix, and $\boldsymbol{\tau}$ contains the actuator forces. The task of the robot can be defined in the servo-constraint form as

$$\boldsymbol{\Gamma}(\mathbf{q}) = \mathbf{p}(\mathbf{q}) - \mathbf{h}(t) = \mathbf{0}, \tag{2}$$

where $\mathbf{p}(\mathbf{q})$ describes the controlled point as function of the generalized coordinates and $\mathbf{h}(t)$ defines the corresponding desired trajectory.
We suppose a coordinate transformation $[\mathbf{y}_a^\intercal \;\; \boldsymbol{\xi}_u^\intercal]^\intercal = \boldsymbol{\phi}(\mathbf{q})$ exists with which the equation of motion (eq. (1)) can be written in the following form:

$$\begin{bmatrix} \tilde{\mathbf{M}}_{aa} & \tilde{\mathbf{M}}_{au} \\ \tilde{\mathbf{M}}_{ua} & \tilde{\mathbf{M}}_{uu} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{y}}_a \\ \ddot{\boldsymbol{\xi}}_u \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{c}}_a \\ \tilde{\mathbf{c}}_u \end{bmatrix} = \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} \boldsymbol{\tau}, \tag{3}$$

where $\mathbf{y}_a$ represents the directions which are directly influenced by the control force $\boldsymbol{\tau}$ and $\boldsymbol{\xi}_u$ represents the non-driven dynamics. From the partitioned equation of motion (eq. (3)) we select the unactuated part which does not contain the input force $\boldsymbol{\tau}$, and we linearize it around the certain preselected configuration along the desired trajectory as

$$\tilde{\mathbf{M}}_{uu}\ddot{\boldsymbol{\xi}}_u + \mathbf{b}_1\dot{\boldsymbol{\xi}}_u + \mathbf{b}_2\boldsymbol{\xi}_u = -\tilde{\mathbf{M}}_{ua}\ddot{\mathbf{y}}_a - \mathbf{b}_3\dot{\mathbf{y}}_a - \mathbf{b}_4\mathbf{y}_a + \mathbf{b}_5, \tag{4}$$

where $\mathbf{b}_i, i = 1...5$ are constant coefficients. Assuming $\mathbf{y}_a$ is known for a segment of the desired motion, the solution of eq. (4) for $\boldsymbol{\xi}_u$ can be determined in closed form and the trajectory tracking error is given by eq. (2). This error depends on the prescribed trajectory of the actuated coordinates and can be minimized via the selections of those. Here we propose to choose $\mathbf{y}_a$ as a polynomial function of time with constant coefficients and based on the closed form solution of eq. (4) minimize the violation of the servo-constrains $\boldsymbol{\Gamma}(\mathbf{q})$ (eq. (2)) in order to determine the desired values for $\mathbf{y}_a$. This can be then used in eq. (3) to calculate the desired actuator forces for each segments of the desired motion.

Numerical results

In order to demonstrate the usability of the method we applied it on a 3 DoF serial planar manipulator. This robot has only 2 actuators while the third joint is passive. This benchmark example is used in several publications [4], [1] as a mechanical model of a flexible manipulator (see: Fig. 1). In the reference [1] it is showed that with the computed torque control the internal dynamics is unstable when the endpoint of the robot is controlled. In order to overcome this problem and to achieve a stable operation instead of the endpoint of the robot the middle of the 3rd arm is controlled. This intuitive modification makes the controlled system stable. The mechanical parameters of the robot and desired task was the same with reference [1]. In the simulation the end-point of the robot performs a rest-to-rest (points $\mathbf{r}_0$ an $\mathbf{r}_f$ in Fig. 1) manoeuvre with a prescribed circular trajectory. The motion of the task is illustrated in Fig. 1. In Fig. 2 the norm of servo constraint violations are depicted in case of the method of [1] and in case of the predictive approach. In case of the reference controller the maximum deviation of the servo-constraints was $\max(|\mathbf{\Gamma}|) = 0.28[\mathrm{m}]$ and the root mean square value for the whole simulation was $\mathrm{RMS} = 0.093[\mathrm{m}]$. In case of the predictive method the maximal value of the servo-constraint violation norm $\max(|\mathbf{\Gamma}|) = 0.13[\mathrm{m}]$ and the root mean square value was $\mathrm{RMS} = 0.045[\mathrm{m}]$.

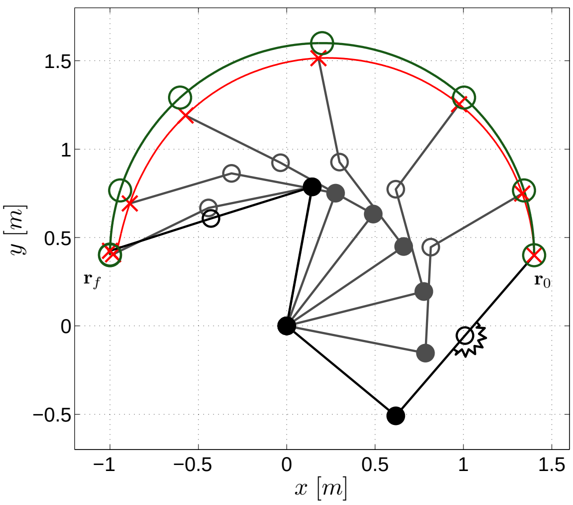

Figure 1: Illustration of the motion

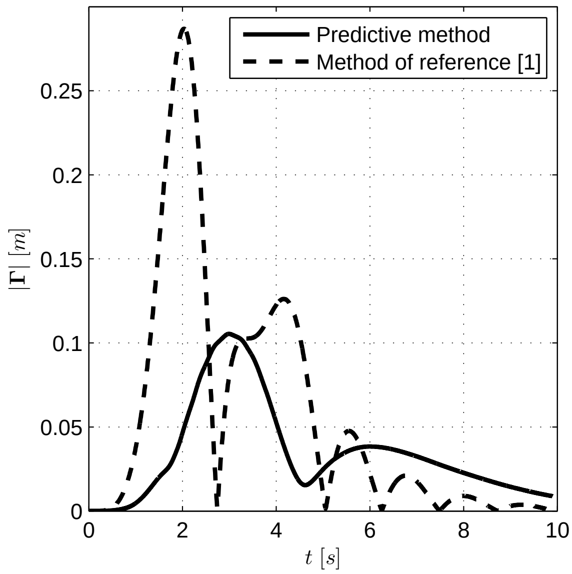

Figure 2: The norm of the servo-constraint violation

Conclusions

It can be stated that the proposed predictive method can be used effectively in case of underactuated manipulators. The advantage of this method is that it does not require an intuitive modification of the servo-constraints. The simulation results show that with the presented method the system could operate in a stable way.
The results are promising, as the proposed predictive control strategy could considerably decrease the error of trajectory tracking compared to the computed torque control technique.

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