Vibration localization and snaking bifurcations in a purely mechanical system

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Summary. Spatially localized states have been encountered in different physics fields. We study the dynamic response of a chain of weakly coupled nonlinear oscillators showing, that snaking bifurcation patterns can be found in purely structural systems, corresponding to localised vibration states. A detailed analysis of the vibration shape and of the energy content of the localized solutions is conducted to show similarities and differences with the known snaking structure.

Introduction

Spatially localised states of dynamical systems have been studied in a large number of different fields in the sciences and in engineering from optics to granular matter. In particular in fluid dynamics spatially localised convection rolls have been observed in water-ethanol mixtures [1]-[2], where localised convection domains of arbitrary length are found to be stable, being surrounded by the conductive state. These localised states in the bifurcation diagram have shown to give birth to what is called a snaking structure [3]. Typically snaking bifurcation diagram involves two snaking solution branches, intertwined into each other, possibly linked by an unstable branch, called "ladder". Although snaking bifurcations are now generally known and studied in many fields of dynamical systems, it seems that there is hardly any study into the phenomenon in the context of structural vibrations in engineering. We choose a simple structural system: a chain of (weakly non-linear) oscillators coupled into a linear oscillator chain where the nonlinearities come from the non-linear damping terms that can be thought of bringing into our purely structural model the corresponding non-linear forcing and dissipation terms from surrounding flow, or an involved friction interface. Interestingly, the results do turn out different to many of the hitherto reported snaking bifurcation patterns as much of the pattern as a whole seems to have disintegrated into isolated branches. Similarities and differences with the classical snaking phenomenon are analyzed.

The mechanical system

We consider a cyclic system of N_{dof} non-linear oscillators, see Figure 1, which are coupled via a weak linear spring of stiffness k_Δ. Each oscillator has mass m and is linked to the ground via a linear spring k and a non-linear damper which introduces a velocity proportional force of the form

$$F_v = -c_1\dot{x} + c_3\dot{x}^3 - c_5\dot{x}^5. \tag{1}$$

Here x denotes the displacement of the individual oscillator, $\dot{x}$ the velocity, and we introduced the coefficients c_1, c_3, c_5 to parametrise the velocity-dependent force.

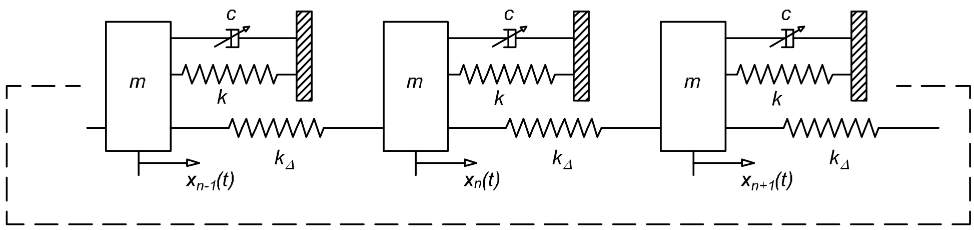

Figure 1. The model system under study.

We introduce the quantities $\omega_0 = \sqrt{k/m}, \eta_\Delta = k_\Delta/k, \xi_i = \frac{c_i}{2\sqrt{km}}, \tau = \omega_0 t$ and write the equilibrium equation in dimensionless form, obtaining

$$q_1\ddot{\widetilde{x}}_n + q_2\dot{\widetilde{x}}_n - q_3\dot{\widetilde{x}}_n^3 + q_4\dot{\widetilde{x}}_n^5 + q_5\widetilde{x}_n - q_6\left(\widetilde{x}_{n+1} + \widetilde{x}_{n-1} - 2\widetilde{x}_n\right) = 0, \tag{2}$$

where

$$q_1 = 1, \quad q_2 = 2\xi_1, \quad q_3 = 2\xi_3\omega_0^2 x_0^2, \quad q_4 = 2\xi_5\omega_0^4 x_0^4, \quad q_5 = 1, \quad q_6 = \eta_\Delta, \tag{3}$$

and the $\widetilde{\square}$ superposed indicates that displacements are dimensionless, $\widetilde{x}(\tau) = x(\tau)/x_0$. In (2) we defined a dimensionless time $\tau = \omega_0 t$, which allows to replace $\frac{d}{dt}$ with $\omega_0\frac{d}{d\tau}$. Notice that we choose $\xi_3, \xi_5 > 0$, thus the third degree term of the velocity-dependent force introduces a destabilizing force into the system, while the fifth degree term tends to stabilize it.

We will choose ξ_1, i.e. the linear damping coefficient, as our primary control parameter in a range from -0.4 to $+0.6$. In this range the velocity dependent force changes its shape in a way such that for low ξ_1 values a negative damping is introduced. We therefore apply the Harmonic Balance Method (HBM) as an efficient numerical technique to obtain an approximation to the steady-state solution of the system coupled with a pseudo archlength continuation scheme to follow the solution branches.

Results

We study a cyclically symmetric chain of $N_{dof} = 12$ non-linear oscillators in the zone where the single oscillators are bistable. The parameters used are:

$$\omega_0 = 2\pi, \quad x_0 = 1, \quad \xi_3 = 0.3, \quad \xi_5 = 0.1, \quad \eta_\Delta = 0.01 \tag{4}$$

The results obtained from time integration were used to derive initial conditions for the continuation algorithm. Figure 2 shows, in the left panel, the sum of the maximum potential energy of each mass $\widetilde{U}_{\max} = \frac{1}{2}\sum_{i=1}^{N} \widetilde{x}^2_{\max,i}$ plotted against the linear damping coefficient ξ_1. Many solutions appear to be entangled, making it almost impossible to distinguish one from the other. Looking more closely at the overall structure created by the superposed solutions we can observe trajectories similar to snake and ladder branches [3] and twelve 'steps' (corresponding to the number of oscillators) can be identified. Each step is labelled with a red circle and the corresponding energy distribution is plotted in the twelve bar plots on the right-hand side of Figure 2. From the bottom to the top, at the first step one mass is moving while the others are more or less motionless, at the second step two masses are moving and so on up to the twelfth. This resembles the usual snaking behaviour, where, for example in fluid dynamics, the steps can be related to the increasing number of convection rolls.

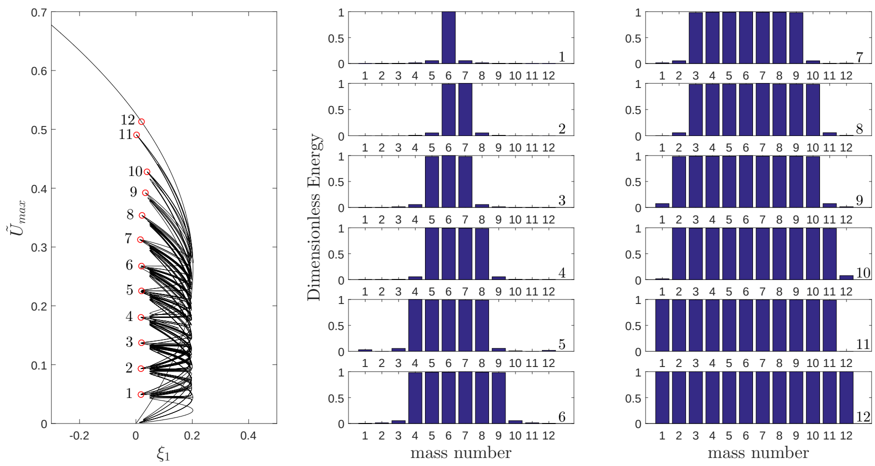

Figure 2. Left: bifurcation diagram for the non-linear oscillator chain. The complex snaking pattern linking the spatially homogeneous stationary static state with the state where all oscillators are vibrating fills the zone of bistability. Middle and Right: the average dimensionless energy of each mass for the 12 equilibrium solutions which are marked with a red circle in the snaking pattern.

Conclusions

In this work we have studied snaking bifurcations of a non-linear cyclically symmetric oscillator chain. The bifurcation diagrams resulting in the bistability zone resemble typical snaking patterns, but also show marked differences. The solution branches are composed of isolas, which have a figure eight shape in the bifurcation diagram. When the isolas are put together, they picture a typical snaking pattern. Still, our findings suggest that the snaking behaviour in structural dynamics could be more complicated due to the superposition of different non-linear mode shapes: solutions which have very different shapes present almost the same energy content and thus the corresponding solution branches overlap or are very close to each other in the bifurcation pattern. From the present results it has become clear that more work on snaking in engineering structures is necessary.

References

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