Forced Vibrations of a String in the Presence of a Smooth Unilateral Obstacle

Harkirat Singh and Pankaj Wahi
Mechanics and Applied Mathematics Group
Department of Mechanical Engineering, Indian Institute of Technology Kanpur, India

Summary. In this study, we investigate the forced vibrations of a string in the presence of a smooth unilateral obstacle. This problem is of central importance to Indian string musical instruments like *Tanpura* and *Sitar*. The mathematical model is developed using Hamilton's principle followed by system discretization using the Galerkin projection approach. In particular, we explore different types of motions in mutually perpendicular directions and study the coupling between them. Finally, we study the stability of periodic motions and investigate the effect of system parameters on the stability.

Introduction

Earlier theoretical studies [1, 2, 3] investigated the free vibrations of a string in the presence of a boundary obstacle. The analysis revealed that only one planar motion is possible. This planar motion is stable for small amplitude of vibrations and it becomes unstable beyond a certain critical amplitude. However, there are numerous studies [4, 5] exploring the nonlinear dynamics for the forced vibrations of a string in the absence of an obstacle. The role of the obstacle on the nonlinear aspects of the string vibrations is still unexplored. In the present study, we investigate the string vibrations with distributed unidirectional periodic force in the presence of an obstacle.

Problem Formulation

A schematic representation of the system under consideration is shown in figure 1. The source of nonlinearity in our system is assumption of variable tension along the string and presence of finite curved obstacle. The mathematical model incorporating non-planar motions and moving boundary is developed. We further assume that the string remains tangent to the obstacle surface at the point of separation ($X = \Gamma(t)$) in figure 1. The equations of motion of our system are nonlinear coupled PDEs (not shown), governing the motions in mutually perpendicular directions, which are discretized using Galerkin projection approach.

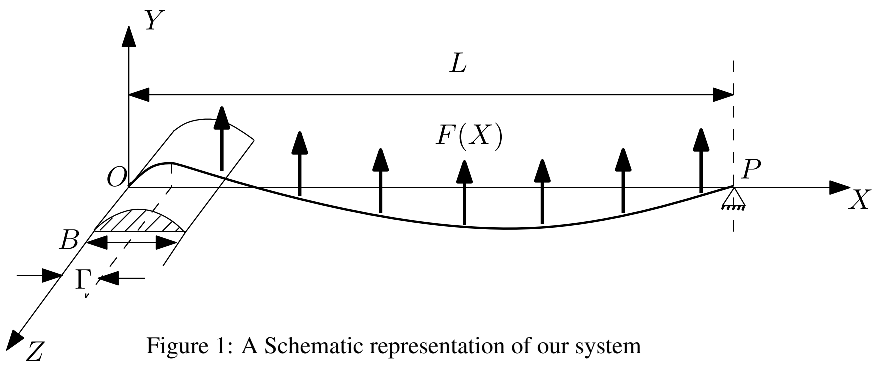

Figure 1: A Schematic representation of our system

Observations

Firstly, we study the string vibrations due to distributed periodic forcing in Y-direction. We observe that the trivial (all zeros) initial condition leads to periodic modulations in the $X-Y$ plane. However, trivial initial condition corresponding to forcing in the Z-direction leads to periodic modulation in both Y and Z-direction. This indicates that there is a possibility of only one planar motion. We also observe that a perturbation perpendicular (α modulation) to the planar motion decays below a certain frequency as shown in figures 2A and 2B. On the contrary, this perturbation grows and reaches a steady state giving rise to periodic nonplanar motions beyond a certain critical frequency as shown in figures 2C and 2D. The value of the critical frequnecy in our case comes out to be 3.36.

Below the critical frequency, the periodic solutions correspond to the planar motions only as shown in figure 3. Above the critical frequency, the periodic solutions correspond to the planar as well as nonplanar motions as shown in figure 3. The planar motions below the critical frequency are stable. This branch of planar motions becomes unstable beyond the critical frequnecy. Moreover, two stable branches of nonplanar motion originate just after the critical frequency. There is a another pair of unstable branches of the nonplanar motions which exists in the region above the critical frequency. In the region beyond the critical frequency, there exists both stable planar branch and stable nonplanar branch.

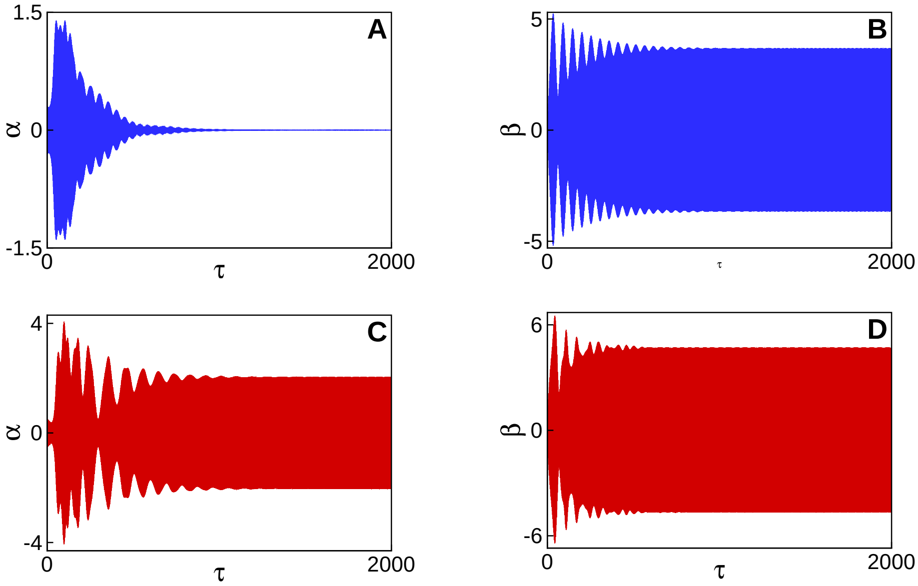

Figure 2: Representation of stability of the planar motion. Figures A and B represent the α and β modulations corresponding to forcing frequency (Ω)= 3.2. Figures C and D represent the α and β modulations corresponding to $\Omega = 3.4$. Note: We have included a very small magnitude of modal damping to eliminate the transient response.

Future Directions

We have discussed the qualitative change in the nature of solutions depending on the forcing frequency keeping the other parameters as constant. Similar analysis can also be performed with different values of other parameters. Furthermore, the effect of friction between the string and the obstacle will be reported in our future study. It will be interesting to determine the response due to distributed forcing in the Z-direction. We will also analyze the string vibrations under periodic force limited over space and time to mimic the actual playing of *Tanpura* and *Sitar*.

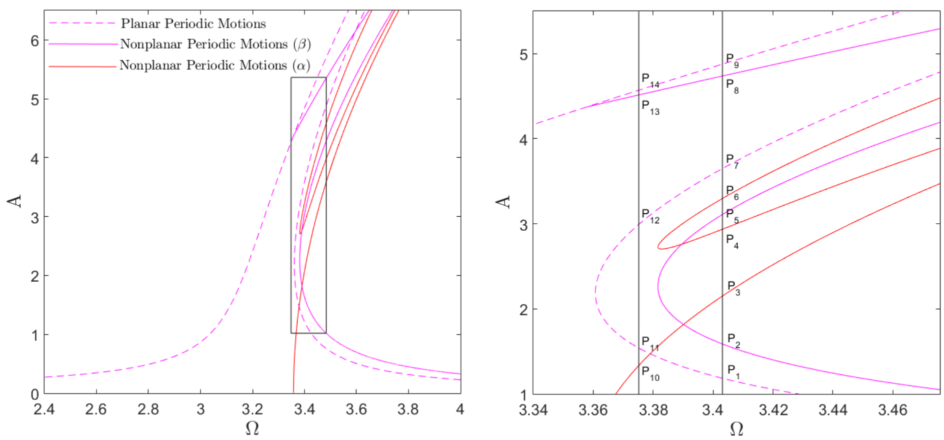

Figure 3: Amplitude versus frequency representation for the periodic solutions. Left: The critical frequency corresponding to the onset of periodic nonplanar solutions is **3.36**. Right: Zoomed view of the rectangular portion demarcated in the left plot. (P_1, 0), (P_3, P_8), (P_{10}, P_{13}), and (P_{11}, 0) are stable solutions. (P_4, P_5), (P_2, P_6), (P_7, 0), (P_9, 0), (P_{12}, 0) and (P_{14}, 0) are unstable solutions.

References

[1] Singh H., Wahi P. (2016) Non-planar vibrations of a string in the presence of a boundary obstacle. *JSV* **389**:326-349.

[2] Singh H., Wahi P. (2017) Role of curvatures in determining the characteristics of a string vibrating against a doubly curved obstacle. *DOI: 10.1016/j.jsv.2017.04.043*.

[3] Mandal A.K., Wahi P. (2015) Natural frequencies, modeshapes and modal interactions for strings vibrating against an obstacle :Relevance to Sitar and Veena. *JSV* **338**:42-59.

[4] Johnson, J.M., Bajaj A.K. (1989) Amplitude modulated and chaotic dynamics in resonant motion of strings. *JSV* **128**:87-107.

[5] Miles, J. (1984) Resonant, nonplanar motion of a stretched string. *JASA* **75**:1505-1510.