Nonlinear Characteristics of Hunting Motion of a Railway Wheel Set By Using a Roller Rig

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Summary. Railway vehicle wheel set experiences the problem of hunting motion above a critical speed, which makes passenger uncomfortable and threatens the safety of railway vehicles. The vehicle even below the critical speed, which is obtained by liner analysis, can experience the hunting motion depending on the initial condition, due to the nonlinear characteristics. In order to investigate the nonlinear characteristics in the hunting motion, a roller rig device is used in this paper. The nontrivial steady state by influence of the quintic nonlinearity is experimentally detected below the linear critical speed in the experiment of the roller rig.

Introduction

Railway vehicle wheel set experiences the problem of hunting motion above a critical speed, which is a kind of self-excited oscillation, due to the creep force between the wheels and the rails [1]. This phenomenon makes passengers uncomfortable, damages the wheels and the rails, even causes a derailment. Some stabilization method have been proposed (for example [2]). A simple model of the wheel set is often formulated to investigate the hunting motion of railway vehicles. The critical speed of hunting motion can be obtained by analyzing the simple model of wheel set by linear theory [3]. In addition, the third order nonlinear analysis reveals that a subcritical Hopf bifurcation is produced at the critical speed. It proves that railway vehicle even below the critical speed can experience the hunting motion depending on the initial condition [4][5]. For this characterization, the nonlinearity should be considered in the wheel system. In the nonlinear analysis, the center manifold theory was employed to analyze the nonlinear characteristics of the bifurcation. In order to experimentally investigate the subcritical hopf bifurcation, a roller rig device is used in this paper. As a result, there are two stable amplitude of a wheel set in a speed range which is below the critical speed and it proves that the fifth order terms play a significant role for the dynamics below the critical speed in wheel system.

Analytical Model and Equation of Motion

The roller rig device and wheel set used in this paper is shown in Fig. 1. The mathematical model has two degrees of freedom, lateral motion y and yaw motion ψ. The running direction is x and running speed v is constant. As shown in Fig. 2, r_0, k_0, γ_0, and d_0 are the centered wheel rolling radius, x direction stiffness, wheel tread angle, and half-track gauge, respectively.

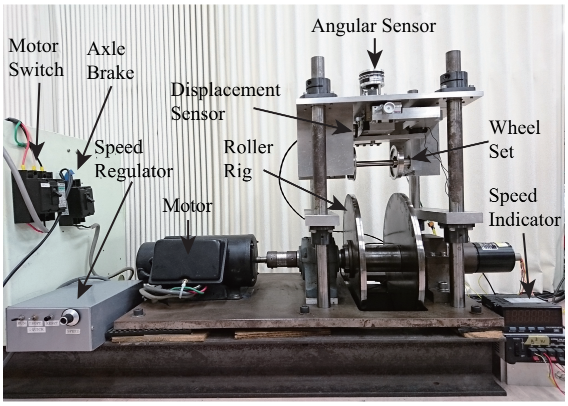

Figure 1: Roller rig device.

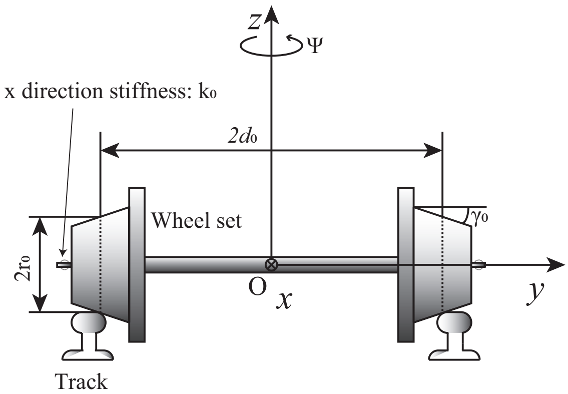

Figure 2: Configuration of the wheel set and rails.

The half-track gauge d_0 denotes the representative length and the inverse value of natural frequency ω_ψ denotes the representative time. The dimensionless equations governing the lateral motion y^* and yaw motion ψ are obtained as follows:

$$\ddot{y}^* + \frac{d'_{11}}{v^*}\dot{y}^* + k_{11}y^* + k_{12}\psi + \alpha_{yyy}y^{*3} + \alpha_{yy\psi}y^{*2}\psi + \alpha_{y\psi\psi}y^*\psi^2 + \alpha_{\psi\psi\psi}\psi^3 + O(5) = 0, \tag{1}$$

$$\ddot{\psi}^* + \frac{d'_{22}}{v^*}\dot{\psi}^* + k_{21}y^* + k_{22}\psi + \beta_{yyy}y^{*3} + \beta_{yy\psi}y^{*2}\psi + \beta_{y\psi\psi}y^*\psi^2 + \beta_{\psi\psi\psi}\psi^3 + O(5) = 0, \tag{2}$$

where the dot denotes the derivative with respect to the dimensionless time t^*, v^* denotes the dimensionless running speed, and $\alpha_{yyy}, ..., \beta_{\psi\psi\psi}$ are cubic nonlinear terms with eight unknown coefficients. The symbol of $O(5)$ denotes the quintic nonlinearity. Not only the cubic nonlinearity but also this quintic nonlinearity plays an important role for the stable nontrivial steady state [6], which is experimentally shown by a roller rig in this study. The elements of the above matrices are expressed as follows:

$$d_{11} = \frac{2\kappa_y}{md_0\omega_\psi^2},\ d_{22} = \frac{2\kappa_x d_0}{I\omega_\psi^2},\ v^* = \frac{v}{d_0 I\omega_\psi^2},\ k_{11} = (1 - \frac{l_0}{l}(\frac{\omega_y}{\omega_\psi})^2),$$

$$k_{22} = 1,\ k_{12} = \frac{-2\kappa_y}{md_0\omega_\psi^2},\ k_{21} = \frac{2d_0^2\kappa_x\gamma_0}{Ir_0\omega_\psi^2}.$$

Here, m, I, l_0 and l represent the mass of the wheel set, the moment of inertia, the natural length of the spring and the length of the spring in the equilibrium state, respectively; κ_x and κ_y are creep coefficients in the x direction and y direction; ω_ψ is the natural frequency of the wheel set in the yaw direction and ω_y is the natural frequency of the wheel set in the lateral direction.

Experiment of Roller Rig

The oscillation of the wheel set under different speed is recorded by using a displacement sensor and a angular sensor. Also, the maximum amplitude of lateral and yaw directions, which observed to be stable, is denoted as a stable amplitude. The stable amplitude of lateral direction, yaw direction and the spectrum map in different speed are down in Figs. 3 and 4.

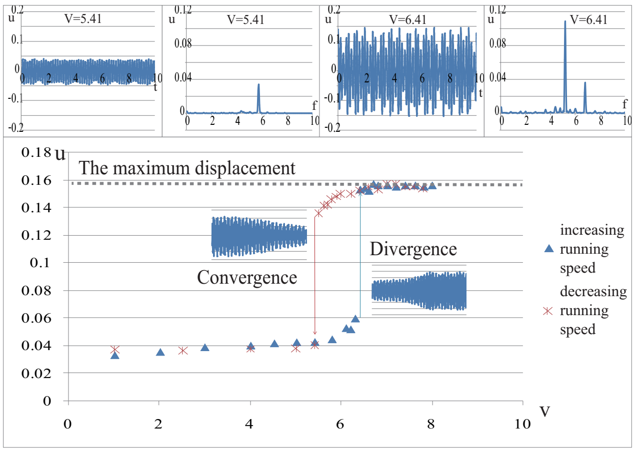

Figure 3: Stable amplitude of lateral direction

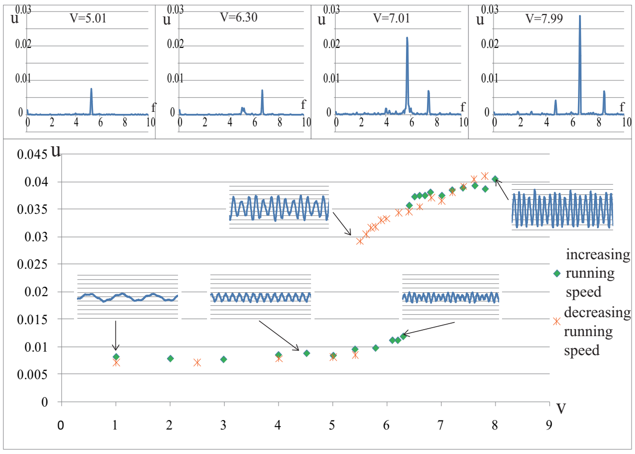

Figure 4: Stable amplitude of yaw direction

Here, the lateral direction has a maximum displacement. The existence of subcritical Hopf and saddle-node due to the cubic and quintic nonlinear effects is experimentally observed by using the roller rig.

Conclusions

The roller rig has been widely applied to the study on hunting motion in particular at the linear critical speed. By using the roller rig, the present study reveals the nonlinearity of the hunting motion produced through the subcritical Hopf bifurcation and the saddle-node one. Fundamental studies on the nonlinearity of hunting motion are expected to be advanced by using the experimental apparatus with a roller rig.

References

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