Compensating symmetry breaking in planetary gearboxes by means of tooth profile modifications

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Summary. Nonlinear vibrations can produce imbalance of dynamic forces on the sun of symmetric planetary gearboxes, this is due to the symmetry breaking and results in strong vibrations of the sun supports.
Changing the bearing stiffness can modify the natural frequencies and consequently the ratio between frequencies; when such ratios are rational numbers the nonlinear system can suffer of nonlinear modal interactions due to internal resonances and complex dynamics. One of the well known consequences of the onset of complex dynamics in symmetric dynamical systems is the symmetry breaking.
It was proven in the past that chaos induced symmetry breaking in planetary gearboxes can generate undesirable loads on the sun supports, which are not predictable using the classical design tools. The present study shows that tooth profile modifications on sun and planet gears can positively affect the dynamic response of the system and minimize the possibility of failure caused by symmetry breaking and unexpected loads on sun bearings.

Introduction

Planetary or epicyclic gear trains are widely used in many automotive, aerospace and marine applications; they are effective power transmission systems when high torque to weight ratios, large speed reductions in compact volumes, co-axial shaft arrangements, high reliability and superior efficiency are required [1]. Gear vibrations are primary concerns in most planetary gear transmission applications, where the manifest problem may be noise or dynamic forces. The most important source of vibration in planetary gears is the parametric excitation due to the periodically time-varying mesh stiffness of each sun-planet and ring-planet gear, because the number of tooth pairs in contact changes during gear rotation. This mesh stiffness variation parametrically excites the planetary gear system, causing severe vibrations when a harmonic component approaches one of the natural frequencies (or their linear combinations). Under certain near resonant operating conditions, gear systems can experience a teeth separation that induces nonlinear effects such as jump phenomena and subharmonic and superharmonic resonances with dramatic effects on the dynamic response [2]. These phenomena have been deeply investigated in geared systems during the last 20 years [3-7].

This paper presents a dynamic model to simulate the dynamic behavior of a single-stage planetary gear system with time varying mesh stiffness and backlash. The complex dynamic scenario of a three-planets gearbox is investigated in detail. A bifurcation analysis is performed to explore the dynamic scenario (periodic, quasiperiodic and chaotic), with a special attention to symmetry breaking phenomena that are extremely interesting in planetary gears as they can cause additional imbalance-induced-stresses. Numerical analyses are carried out over meaningful mesh frequency ranges. The analysis is completed with time histories, spectra, phase portraits and Poincaré maps of the most interesting regimes. It was proven in the past that chaos induced symmetry breaking in symmetric planetary gearboxes can generate unexpected and undesirable loads on the sun supports, which are not predictable using the classical design tools. The present study shows that tooth profile modifications on sun and planet gears can positively affect the dynamic response of the system and minimize the possibility of failure caused by symmetry breaking and unexpected loads on sun bearings.

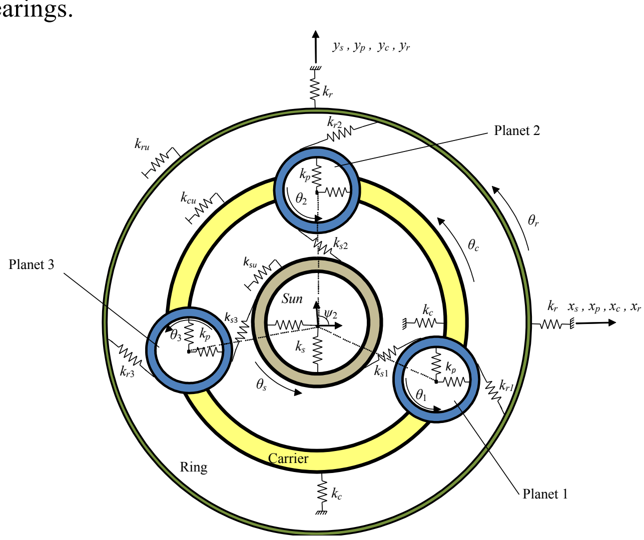

Fig. 1. Physical planar model of the single stage planetary gearbox

Dynamical Model

The physical model of a single-stage planetary gear set is shown in Fig. 1. The system is made of four types of elements: sun gear; ring gear; N planets; carrier. Here the modeling is plane, i.e. each element has three degrees of freedom: two displacements and a rotation. The centers of the different elements of the system are free to move in the plane, each component has translational and rotational degrees of freedom. The total number of degrees of freedom is (3N + 9); the model includes time variation of gear mesh stiffness (depending on the reciprocal angular position of two meshing gears), backlash nonlinearities of mating gears and bearing compliance (no clearance is considered for bearings). The basic dynamical equilibrium equations contain (3N+9) nonlinear ordinary differential equations, where N is the number of planets; e.g. when N=3 they will be 18 coupled equations. The equations of motion of the model shown in Fig. 1 are written using Newton-Euler equations and they are placed in canonical form.
For the sake of brevity only the Sun Gear equation of motion is reported here:

$$
\begin{aligned}
&-M_s \times \ddot{x}_s - C_s \times \dot{x}_s + \sum_{n=1}^{N}\Big[\big(c_{sn}(\dot{x}_n - \dot{x}_s)\times \sin^2(\psi_n - \alpha_s)\big) + \big(-c_{sn}(\dot{y}_n - \dot{y}_s)\times \sin(\psi_n - \alpha_s)\times \cos(\psi_n - \alpha_s)\big) \\
&+\big(c_{sn}(\dot{\theta}_s \times r_{bs} + \dot{\theta}_n \times r_{bn})\times \sin(\psi_n - \alpha_s)\big)\Big] - k_s \times x_s + \sum_{n=1}^{N}\Big[\big(k_{sn}(f_{sx})\times \sin^2(\psi_n - \alpha_s)\big) + \big(-k_{sn}(f_{sy})\times \sin(\psi_n - \alpha_s)\times \cos(\psi_n - \alpha_s)\big) + \big(k_{sn}(f_{s\theta})\times \sin(\psi_n - \alpha_s)\big)\Big] = 0
\end{aligned}
\qquad 1)
$$

the piecewise-linear displacement functions for sun-planets meshing, which are defined as follows:

$$
f_{sx} = \begin{cases} x_n - x_s - \dfrac{b_s}{\sin(\psi_n - \alpha_s)} & \Delta_{sn} \geq b_s \\ 0 & |\Delta_{sn}| < b_s \\ x_n - x_s + \dfrac{b_s}{\sin(\psi_n - \alpha_s)} & \Delta_{sn} \leq -b_s \end{cases}
\qquad 2)
$$

Dynamical Model

Results are obtained by direct numerical integration using an implicit Runge-Kutta scheme (RADAU). Figures 2 represent the bifurcation diagram of the Poincaré maps, they correspond to the x and y translations of sun gear center. In linear field, the symmetry and the perfect balancing of the system implies that the sun is loaded with a self-equilibrate force system from the planets, therefore it should experience no displacement in x and y directions. For the ranges where complex phenomena appear, the system presents symmetry breaking and consequently sun imbalance.

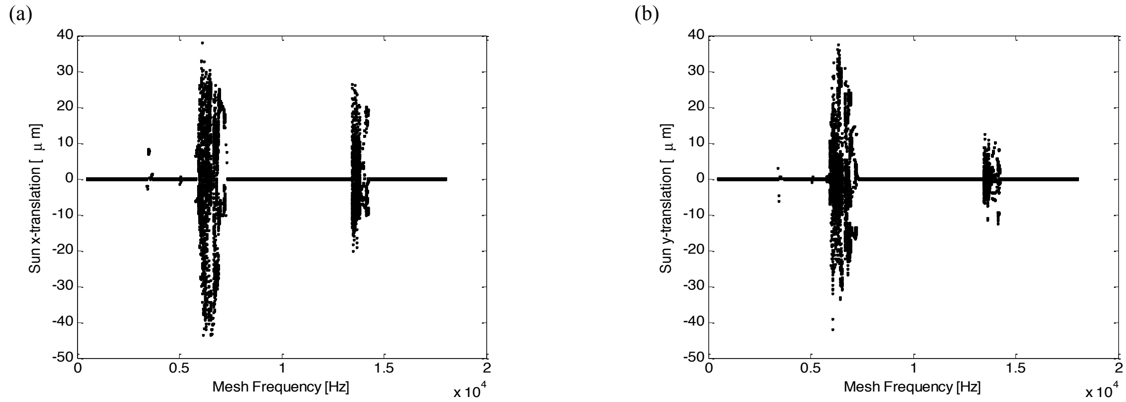

Fig. 2. Bifurcation diagram vs. mesh frequency for case 2 (a) Sun center x-translation [μm], (b) Sun center y-translation [μm]

Conclusions

This paper presents a dynamic model to simulate the dynamic behavior of a single-stage planetary gear system with time varying mesh stiffness and backlash. The bifurcation scenario shows a symmetry breaking phenomenon that can cause additional imbalance-induced-stresses. A compensation can be achieved using suitable profile modifications.

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