Preloading in Nonlinear Oscillator

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Summary. In this paper the influence of the preloading force on the vibration of the nonlinear oscillator is investigated. In the real oscillatory systems with significant deformation where the restitution force is a strong nonlinear function, the action of the preloading force has to be taken into consideration. Namely, measuring on nonlinear systems gives the rigidity coefficients which depend on the preloading force. Using these coefficients and the nonlinear property of the elastic force the strong nonlinear oscillator equation is formed. The nonlinearity is of integer or non-integer order where the order of nonlinearity depends on the preloading, too. The solution of the equation is obtained in the form of the ca-Ateb function. The frequency and the period of vibration property of the oscillator are determined. The analytical solutions are compared with numerical ones. They are in good agreement. The obtained results are applicable in building structures but also in systems for energy harvesting or flight control.

Introduction

In the real oscillatory systems, like building structures, the preloading force exists. Usually, this force is neglected due to the fact that the system is assumed to have small deformation and is considered as a linear one. In the systems with significant deformation, the preloading cannot be omitted. Namely, as it is already shown in papers [1] and [2] the restitution force for high deformation is a nonlinear function and the preloading has to be included into considered. Experiments done on such nonlinear oscillators give the coefficients of the restitution force which depend on the preloading. The suggested restitution force in [3] has the form

$$F_r^{eksp} = c(T)\,\dot{x} + k_{lin}(T)\,x + k_{nl}(T)\,|x|^{\alpha(T)}\,\mathrm{sgn}(x) \qquad (1)$$

where the coefficients c, k_{lin}, k_{nl} are functions of preloading T. The order of nonlinearity $\alpha \in R$ is also the function of the preloading T. Based on the experimentally obtained discrete results using the fitting procedure the mathematical functions $c(T)$, $k_{lin}(T)$, $k_{nl}(T)$ and $\alpha(T)$ are formulated as:

$$c(T) = 1.02 + 1.4810^{-7}\exp(1.48T), \quad k_{lin}(T) = 86.94T - 251.03,$$

$$k_{nl}(T) = -47.52T^2 + 495.2T - 150, \quad \alpha(T) = 3.35T^2 - 0.1T - 0.0016$$

For the restitution force (1) the model of the oscillator is as follows

$$\ddot{x} + c(T)\,\dot{x} + k_{lin}(T)\,x + k_{nl}(T)\,|x|^{\alpha(T)}\,\mathrm{sgn}(x) = 0 \qquad (2)$$

It is a strong nonlinear differential equation. Analyzing the coefficients of equation (2) obtained by measuring it is concluded that the pure nonlinear term is much more significant than the linear elastic and the damping term. Then, the equation (2) is rewritten as

$$\ddot{x} + k_{nl}(T)\,|x|^{\alpha(T)}\,\mathrm{sgn}(x) = -\varepsilon\, c(T)\,\dot{x} - \varepsilon\, k_{lin}(T)\,x \qquad (3)$$

where ε<<1 is a small parameter. The aim of the paper is to solve the equation (3) as the function of preloading and to analyze the obtained results.

Analytical solving procedure

Relation (3) is assumed as the perturbed version of the pure nonlinear differential equation

$$\ddot{x} + k_{nl}(T)\,|x|^{\alpha(T)}\,\mathrm{sgn}(x) = 0 \qquad (4)$$

For initial conditions $x(0) = A,\ \dot{x}(0) = 0$ the equation (4) has the exact analytical form [4]

$$x = Aca(\alpha, 1, A^{(\alpha(T)-1)/2}\sqrt{(\alpha+1)\frac{k_{nl}(T)}{2}} \qquad (5)$$

where ca is the cosine Ateb function [5]. Based on (5) the exact period of vibration of the oscillator (4) is [6]

$$T_{ex} = \frac{4B\left(\frac{1}{\alpha+1}, \frac{1}{2}\right)}{\sqrt{2(\alpha+1)k_{nl}(T)}}\,|A|^{(1-\alpha(T))/2} \qquad (6)$$

where B is the beta function. Solution of (4) is assumed as the perturbed version of the solution (5)

$$x = A(t)ca(\alpha, 1, A(t)^{(\alpha(T)-1)/2}\sqrt{(\alpha+1)\frac{k_{nl}(T)}{2}} + \theta(t) \qquad (7)$$

where A(t) and θ(t) are unknown time functions. Using the method of variable amplitude and phase adopted for (4) and described in [7] the equation (4) is rewritten into two coupled first order differential equations. Introducing the averaging procedure the approximate solution for amplitude $A(t)$ and phase $\theta(t)$ are obtained. These functions depend on the coefficients of the equation, i.e., on the preloading parameter T.
For the special case when for the preloading $T = T_0$ the coefficient of linear term satisfies the relation $k_{\text{lin}} = 2c^2 (1+\alpha)/(3+\alpha)$ the exact analytical solution is

$$x = A(\exp(-\frac{2ct}{3+\alpha})ca(\alpha,1, A^{(\alpha-1)/2}\frac{3+\alpha}{\alpha-1}\sqrt{(\alpha+1)\frac{k_{nl}(T_0)}{2c^2}}(1-\exp(-\frac{ct(\alpha-1)}{3+\alpha}))+\theta(t)). \quad (8)$$

The amplitude and period functions are

$$A_d = A\exp(-\frac{2ct}{3+\alpha}), \quad T_d = T_{ex}\exp(\frac{c(\alpha-1)t}{3+\alpha}). \quad (9)$$

The amplitude of vibration has the tendency of decrease, while for $\alpha > 1$ the period of vibration increases.
To prove the analytically obtained solutions the numerical simulation of the system for various values of preloading is done.

Main Results

Two groups of problems are considered: one, when the damping is neglected, and second, when the linear viscous damping is taken into account. For the case when the damping is neglected the amplitude of vibration is independent on the initial preloading, but it has an influence on the frequency of vibration. Using the relation (6) and the numerical simulation of the results it is concluded that the period of vibration has a tendency of increase by increasing of the preloading. For certain preloading the period of vibration is longer for higher values of initial amplitude. For the case when damping acts the amplitude-time curve for various values of preloading is plotted in Fig.1. It is evident that the amplitude of vibration decreases in time. The amplitude decrease is faster for higher preloading.

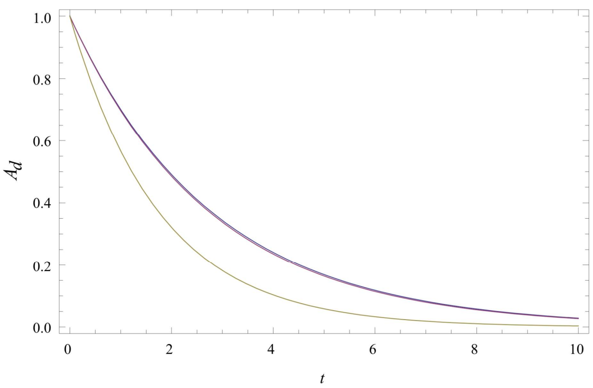

Fig.1. Amplitude-time function for the damping oscillator for preloading of: T=5 lb (blue line), T=7.5 lb (magenta line) and T=10lb (green line).

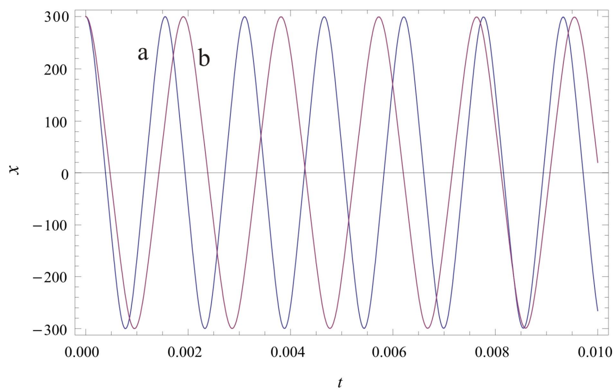

Fig.2. x-t diagrams for the damped system with various preloading: a) T=5 lb (blue line), b) T=7.5 lb (red line).

In Fig.2 the solution of equation (2) for various preloading force is plotted. It can be concluded that the period of vibration is higher for larger preloading. Even for small change of the preloading force the frequency of vibration is shorten significantly.
The obtained results are of interest for designing of building structures with preloading force, but also in other fields of technique like energy harvesting systems, systems for flight control etc.

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