Frictional passive damping in a beam on foundation under moving loads

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Summary. The presentation addresses the dynamic behaviour of frictionally damped Euler-Bernoulli beams on Winkler foundations submitted to a concentrated uniformly moving load. Frictional damping devices are considered to link certain cross sections of the beam to motionless points. A method designed to deal with systems having nonsmooth constitutive laws is used to find numerical solutions [1].

Introduction

The safety of railway vehicles depend in a high degree on the ability to dampen rail amplitudes upon the passage of loads. The purpose of the present study is to quantify how much frictional dissipation (i) affects moving load's critical velocity and (ii) is able to limit dynamic displacements. These are two important aspects, especially for high speed trains that may induce excessive vibrations [2] able to endanger the safety of passengers or at least to make maintenance operations very frequent and expensive. One form of limiting dynamic amplitudes is by means of frictional damping which has some ecological and economic advantages [3] with respect to the more classic viscous damping.

The physical and numerical models

The physical model consist of a Euler-Bernoulli beam of length L, mass per unit length ρ, cross section area A and moment of inertia I, simply supported at both extremities and continuously supported along the span by a Winkler foundation of stiffness per unit length k (Figure 1). The beam (actually a UIC60 rail) is also attached to the immovable part of the

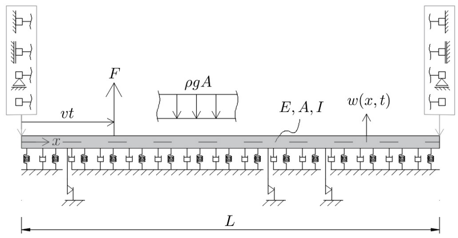


Figure 1: An Euler-Bernoulli beam on a Winkler viscoelastic foundation in parallel with a system of frictional damping devices of the Coulomb type.

foundation by n_{d} frictional dampers obeying to Coulomb's law, each one with a maximum friction force of F_u, located at abcissas x_i. A transverse concentrated load F moves along the beam's axis uniformly with velocity v. The motion is governed by a partial differential inclusion

$$\rho A\ddot{w} + EIw'''' + kw \in -\rho g A + F\delta(x - vt) - F_u \sum_{i=1}^{n_{\mathrm{d}}} \delta(x - x_i)\,\mathrm{Sign}(\dot{w}(x_i, t)), \quad (1)$$

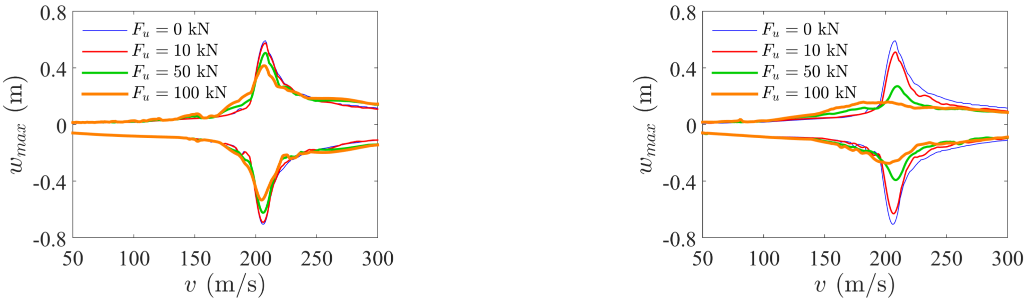
where $w(x,t)$ denotes the transverse displacement at abcissa x in instant t, $\dot{}$ and $'$ denote first order derivatives with respect to t and to x, $\delta(x - x_i)$ denotes the Dirac's function centered at abcissa x_i and the multiapplication $\mathrm{Sign}(z)$ is equal to (a) -1 for $z < 0$, (b) $[-1, +1]$ for $z = 0$, (c) $+1$ for $z > 0$.
The spacial discretization of (1) by the finite element method leads to the ordinary differential inclusion

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} \in \boldsymbol{P} + F\boldsymbol{\Psi}^{\mathrm{T}}(vt) + \boldsymbol{R}(\dot{\boldsymbol{q}}), \quad (2)$$

in which $\boldsymbol{q}$ groups the set of generalized coordinates that are not subjected to any kinematic constraint, $\boldsymbol{M}$ and $\boldsymbol{K}$ are the mass and stiffness matrices, $\boldsymbol{P}$ and $F\boldsymbol{\Psi}^{\mathrm{T}}(vt)$ are the vectors of generalized forces that are statically equivalent to the beam's self weight and to the concentrated moving load and $\boldsymbol{R}$ is the velocity dependent vector of generalized forces statically equivalent to the action of the frictional dampers on the beam.
Numerical approximations to (1) are obtained by applying to (2) a simplified version (for persistent contact with friction) of the nonsmooth contact dynamics method (NSCD) developed by J.J. Moreau [1]. This method is based on the impulsive version of (2).

Numerical results

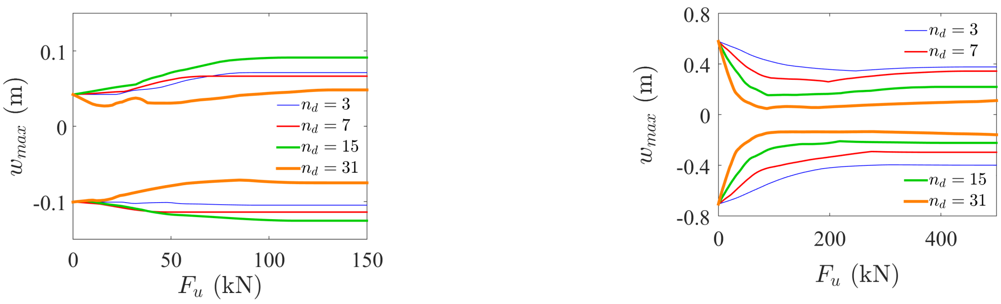
We consider a concentrated force $F = -83.4$ kN traveling on a 200 m UIC60 rail discretized in 256 finite elements. The maximum upward and downward displacements occurring during the time interval $[0, \frac{L}{v}]$ of the load passage are registered in Figure 2 for velocities ranging between 50 m/s and 300 m/s and for a relatively soft foundation ($k = 250$ kN/m^2). Four values of maximum frictional force in each damping device were considered: $F_u = 0$, 10, 50 and 100 kN. Figure 2(a) corresponds to a damping design with a damper at each 50 m while Figure 2(b) corresponds to a design with a damper at each 12.5 m. Such as for beams on viscoelastic foundations, for friction damped beams we identify as



(a) Three equally spaced dampers. (b) Fifteen equally spaced dampers.

Figure 2: Maximum transverse displacements of the beam as function of the velocity of the load for a foundation modulus $k = 250$ kN/m^2 and two numbers of frictional dampers ($n_{\mathrm{d}} = 3$ and 15).

well a load velocity in the vicinity of which larger amplifications of the beam's transverse displacements occur - the so called critical velocity. We observe that for a fixed number of dampers the increase of the dampers' maximal force leads to (i) a decrease of the maximum displacements for velocities in the neighbourhood of the critical velocity, with more intensity when there are more dampers (Figure2), (ii) for a small number of dampers, for subcritical and supercritical load velocities the increase of the maximal force F_u does not contribute to the limitation of the maximal displacements and sometimes (v between 160 and 200 m/s and $v > 240$ m/s) leads to their increase (Figure 2(a)) and (iii) when there is a large number of friction dampers, for supercritical velocities an increase of F_u leads to a decrease of the maximal displacements as for near critical velocities while for subcritical velocities the range of velocities for which an increase of F_u leads to an increase of maximal displacements gets larger (Figure2(b) for v between around 120 m/s and 190 m/s).
Figure 3 illustrates the dependence of the maximal displacements with respect to the maximum friction force in the dampers (F_u) in order to check if, for a given load velocity and a fixed number of damping devices, one may identify an optimal value for the maximum frictional force F_u. We conclude that only for the critical velocity (and certainly for near critical velocities too) it is possible to identify values of F_u corresponding to the minimization of the maximal displacements (Figure 3(b)); moreover, for a fixed F_u, the decrease of the maximal displacement depends monotonically on the number of devices.



(a) Subcritical velocity $v = 150$ m/s. (b) Critical velocity $v = 206$ m/s.

Figure 3: Maximum transverse displacements of the beam as function of the frictional force F_u for a foundation modulus $k = 250$ kN/m^2, two load velocities and different quantities of frictional dampers ($n_{\mathrm{d}} = 3$, 7, 15 and 31).

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