The influence of imperfections on the spatio-temporal dynamics of a parametrically excited nonlinear viscoelastic micro-beam-string

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Summary. A parametrically excited visco-elastic micro-beam-string in an asymmetric dual gap configuration is considered asymptotically and numerically to investigate of the influence of imperfections on its spatio-temporal dynamics. A reconstitution multiple-scales analysis near a one-to-one internal resonance reveals a bifurcation structure that includes coexisting solutions and conditions for orbital loss of stability of a period-doubled solution culminating with non-stationary dynamics.

Nano- and micro-mechanical resonators include various configurations and structural elements which are electromechanically excited at one of their resonance frequencies. Applications of micro-beam resonators include atomic force microscopy [1] and mass sensing [2]. These applications have been implemented for both single element and multi-element arrays [3] that are subject to external [4] or parametric excitation [5]. To-date the majority of theoretical studies assume that electrodes and beams in single element devices and in corresponding multi-element arrays, are equally spaced. We thus assume in this investigation an imperfect configuration with unequal gaps and unequal parametric excitations (Fig.1) and study the spatio-temporal effects of the asymmetrical gap configuration on the nonlinear micro-beam-string response.

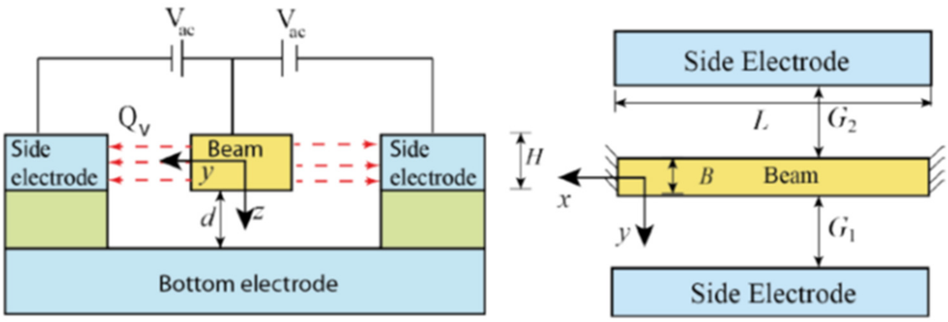

Figure 1: Definition sketch of the micro-beam-string: side view (left), top view (right).

We derive a continuum-based model assuming an asymmetrical electrode gap configuration and nonlinear geometric properties of a visco-elastic micro-beam-string. The resulting nonlinear initial-boundary-value-problem (IBVP) consists of a pair of coupled integro-differential partial differential equations that incorporate cubic elastic membrane stiffness and its corresponding nonlinear visco-elastic dissipation [4]. The micro-beam-string is excited parametrically by unequal components that differ in their amplitudes and phases. A Galerkin methodology is applied directly to the IBVP after pre-multiplication of the visco-elastic fields by the denominator of the generalized force induced by the electro-dynamic excitation [5]. The resulting modal dynamical system consists of a strongly nonlinear parametrically excited modal dynamical system that incorporates cubically coupled in-plane and out-of-plane dynamics.

This system is governed by a one-to-one internal resonance due to the strong pretension of the beam-string where the magnitude of the detuning is governed by the ratio between the width and the thickness of the micro-beam-string. Due to the strong parametric excitation, we apply the reconstitution asymptotic multiple-scales method near the system principle parametric resonance which reveals an intricate bifurcation structure that includes coexisting periodic in-plane and out-of-plane solutions, Stability analysis of the resulting slowly-varying evolution equations enables comparison of the symmetrical and asymmetrical configurations to yield the effect on the micro-beam-string response which includes four primary bifurcation regimes (Fig.2) that are characteristic of parametric excitation with a coupled cubic elastic and dissipative terms: Regions I and IV exhibit a unique and stable trivial solution whereas region II includes and unstable trivial solution and a stable period-doubled solution. Region III portrays bi-stability where a stable trivial solution coexists with a stale period-doubled solution. We note that the band-widths of region II and III are governed by the linear and cubic components of the viscoelastic damping respectively. Thus, models without nonlinear damping exhibit spurious growth without bound of both stable and unstable period-doubled solutions. Furthermore, a secondary bifurcation structure appears in region III which is governed by the magnitude of the imperfection above a critical threshold of excitation.

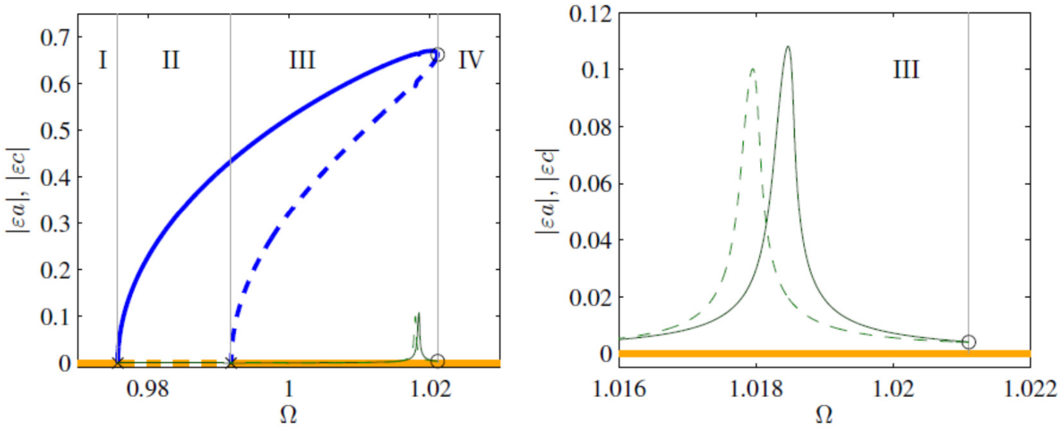

Figure 2: Frequency response near the principle parametric resonance (left) and magnification lower maxima in region III (right).

Numerical analyses of the dynamical system verify the existence of stable period-doubled solutions (Fig.3 left) and demonstrate that the asymmetric configuration can give birth to nonstationary chaotic-like (Fig.3-right) solutions that can coexist with stable periodic solutions beyond a secondary Hopf threshold.

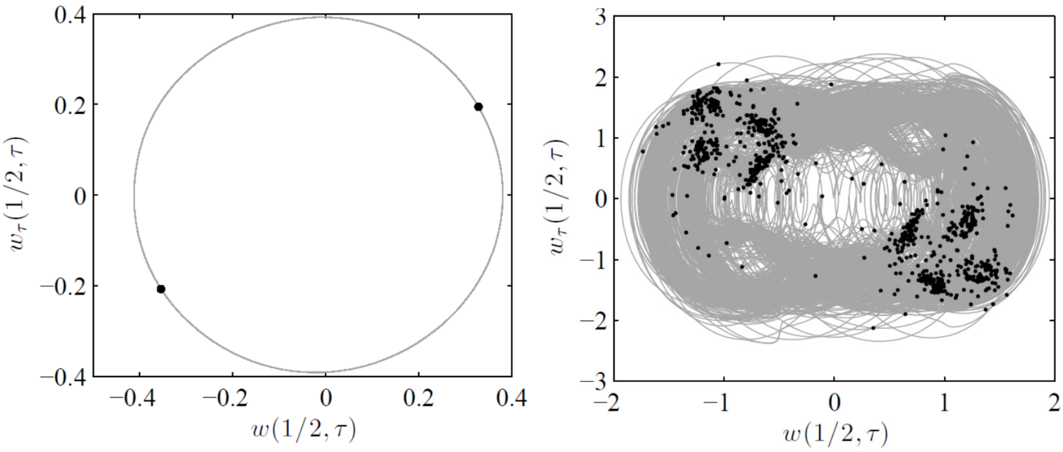

Figure 3: Period-doubled (left) and chaotic (right) dynamics.

Thus, analysis of a nonlinear parametrically excited micro-beam-string in an asymmetric dual gap configuration yields conditions for energy transfer near a one-to-one internal resonance that can be utilized for resonator design for multi-functional sensing and actuation [1].

References

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