A numerical study of elastic Fano resonances in degeneracy-broken trapped mode resonators for biosensing applications

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Summary. This paper presents theoretical, numerical and experimental work, arising in the context of biosensor development, on the existence and characterisation of trapped shear modes of a cyclically symmetric double mesa structure in an elastic infinite plate with all surfaces traction free. These modes are localised due to geometric effects, and as such constitute an isolated point embedded within the continuous spectrum of plane waves of the substrate. By breaking the symmetry of the device, the localisation is weakly destroyed and the resonant profile takes on the characteristic form of a Fano resonance. A numerical study of this novel phenomenon is presented, and the results are discussed with regard to potential applications and interesting mathematical analogies to modern topological phenomena in fluid and quantum mechanics.

Introduction and motivation

A possible candidate for a transformative mechanical resonator for mass spectrometry and biosensing applications is the cyclically symmetric Trapped Mode (TM) structure on a plate. In essence, it consists of a circular raised mesa on an ideally semi-infinite planar surface of an elastic continuum. One of the modes is illustrated in Fig.1 below. . In a separate work presented at this conference, a mathematical theory of the device is developed and presented using a Boundary Integral Equation formulation and solved via a Galerkin method. It is shown therein that the spectrum of the operator contains discrete values embedded in a continuous interval – the mathematical hallmark of true trapped modes.

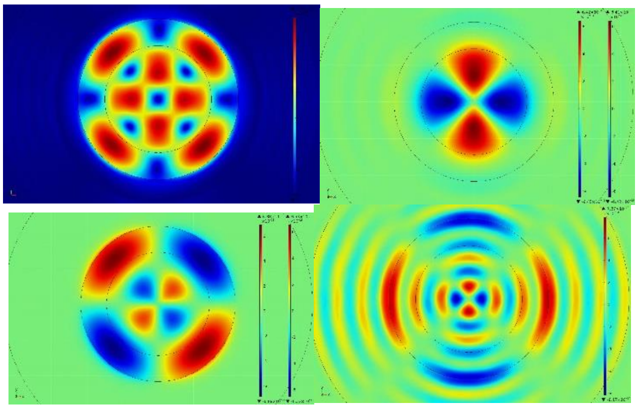

Figure 1: Mode shape of a TM resonator. Top left to bottom right: total displacement; radial, circumferential, and transverse displacements.

The Fano resonance is a physical phenomenon that was first studied in the context of electron scattering in helium[1]. It consists in interference between localised and delocalised wave modes. It gives rise to an asymmetric form for the scattering response **A** different from the more familiar Lorentzian form and given by Fano as:

$$\mathbf{A} = \frac{(\varepsilon + q)^2}{\varepsilon^2 + 1}$$

Where ε characterises the frequency difference between the incident planewaves and the resonant frequency of the discrete state and q is a phenomenological parameter, describing how strongly the profile differs from the Lorentzian form. When q tends to zero, Fano's formula reduces to the Lorentzian.

This work focuses on an interesting phenomenon that emerged during analytical and numerical characterisation of the TM device class. In particular, it was observed that symmetry breaking of the structure by means of a geometric perturbation couples the modal vibration to propagating waves in the substrate. This leads to a Fano-type resonance profile for the forced response that shows very sharp resonance associated with the trapped mode, but which allows for detection using plane-crested acoustic waves and corresponding classical IDT designs. When we consider a periodic, finite array of such devices, the situation becomes one of symmetry-broken Mie scattering. The potential applications and interesting physical connections and analogies are expounded upon, and further work is discussed..

Methods and results

The equations of motion for an isotropic linear elastic solid are presented and applied to the geometry. This is attacked by using the Green's function to write a direct Boundary Integral Equation, which is subsequently solved using a spectral method. The results indicate the presence of a trapped mode, which is decoupled by symmetry from propagating modes in the substrate. The situation is simulated in COMSOL Multiphysics™ via frequency-domain 3D FEA of the elastic structure terminated by PML and fully coupled to an acoustic fluid domain to model the effect of operation in a fluidised environment. Results for frequency response to plane wave and solenoidal excitation of the structure are given, as well as dynamical displacements and pressures in the elastic and fluid continua respectively. The structure is then symmetry broken by the introduction of a penny-shaped crack and the simulation is repeated. The Fano phenomenon is illustrated and discussed.

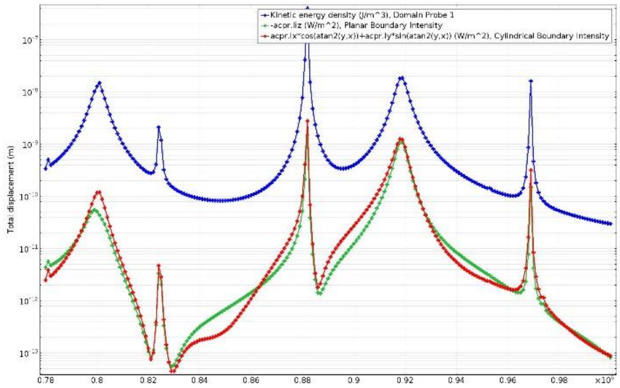

Figure 1. Simulated resonant response profile, showing the characteristic asymmetric Fano form. Markers indicate data points. The blue points show strain energy in the elastic resonator; the red and green points show loss contributions per cycle via fluid radiation.

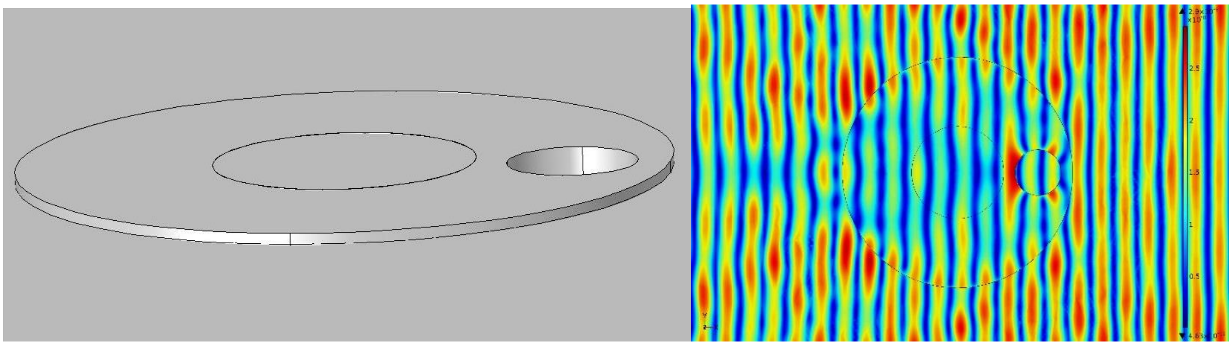

Figure 2. Left to right, simulated trapped mode structure geometrywith penny-shaped crack to break the symmetry and interaction with plane waves.

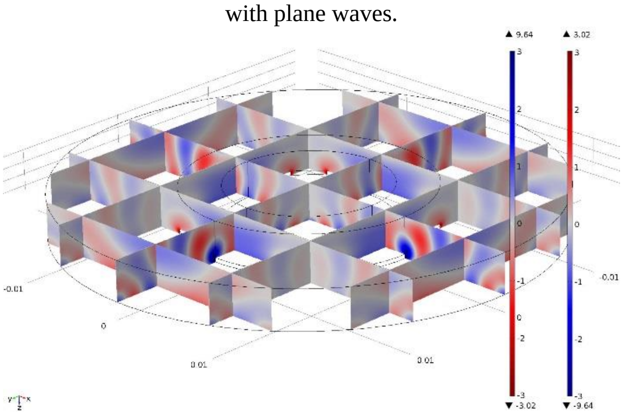

Figure 3. Simulated sound pressure levels in fluid overlayer.

Conclusions and further work

The novel presented mechanism is shown to have significant potential as a biosensor for ultrasensitive detection of pathogens and biomarkers in whole blood. Fascinating connections between the phenomenon of elastic mode trapping and other vortex structures are presented and discussed. Further work of interest, both in pursuing the theoretical connections and in exploiting the technology, is briefly covered.

References

[1] **Fano**, **U.** (1961) Effects of Configuration Interaction on Intensities and Phase Shifts. Phys. Rev **124**:1866–1878