Feedback PD$^{\alpha}$ type Iterative Learning Control for Fractional-Order Human Arm-Support Nonlinear System

Mihailo Lazarević[*], Nikola Đurović[*], Milan Cajić, Boško Cvetković[*], Petar D.Mandić[*] ,Ljubiša Bučanović[**],

[*] *University of Belgrade, Faculty of Mechanical Engineering, Department of Mechanics, Serbia*

[**] *Messer Tehnogas AD, Belgrade, Serbia*

Summary. This paper addresses the problem of application of fractional order Iterative learning control (FOILC) for complex human arm-support system which can be modelled as nonlinear singular system of fractional order. An algorithm of a new strategy for the FOILC implementation is proposed.Sufficient conditions for the convergence in the time domain of a proposed ILC schemes are given by the corresponding theorems and proved. Finally, a numerical simulations show the feasibility and effectiveness of the proposed approach.

Introduction

Iterative learning control (ILC) is one of the recent topics in control theories which belongs to the intelligent control methodology, iteratively improves the behavior of processes that are repetitive in nature, [1]. The basic idea of ILC scheme is, emulating human learning, to use information from previous operation in an attempt to improve performance in a finite time interval from repetition to repetition in the sense that the tracking error is sequentially reduced to zero. In that way ILC is the most suitable for repetitive dynamics and as opposed to traditional controllers, it has unique features such as the structural simplicity, the perfect output tracking, delay compensation, and requires less a priori knowledge about the controlled system in the controller design phase as well as less computational effort than many other kinds of control. In terms of how to use tracking error signal of the previous iteration to form the control signal of the current iteration, ILC updating schemes can be classified as P-type, D-type, PD-type, and PID type.In real application, an ILC scheme is usually performed together with a proper feedback controller for compensation, where we often design a learning operator for the closed-loop (on-line ILC) systems that have achieved a good performance. Since the theories and learning algorithms on ILC were firstly proposed, ILC has attracted considerable interests [2] due to its simplicity and effectiveness of the learning algorithm, and its ability to deal with the problems associated with nonlinear, time-delay, uncertainties, and, recently, singular systems. During the past years, singular systems have attracted attention of a lot of researchers and these systems play important roles in modelling and analysing a lot of practical systems, such as such as robotic systems [3], mechanical systems, electrical circuits, [4] etc. Namely, theory of singular systems shown to be very useful in describing of various dynamical systems because one can obtain more precisely models for the realization of different practical systems, [4]. From the control point of view, it is also necessary to study the ILC algorithms for singular systems, which have repetitive dynamics. Also, increasing attentions are paid to fractional calculus (FC) and its applications in various science and engineering fields, [5], and recently, in control issues and modeling of fractional order singular systems [6]. Moreover, the application of ILC to the fractional-order system has become a new topic, [7,8] the existing fractional-order ILC methods for fractional-order systems only focus on the non-singular systems. For the first time, ILC for fractional order singular systems is suggested in paper [9], where it is introduced robust feedback second order ILC control for a given class of uncertain fractional order singular system.

In this work, we will study ILC control issues for a dynamic model of the arm-support system (robot-assisted therapy) modelled as nonlinear system, to solve the trajectory tracking problem of given nonlinear system. In addition, for the new fractional derivative model of this kind of singular system, [10], feedback PD$^{\alpha}$ type of fractional order ILC will be applied and examined for changes of gain parameters of proposed ILC scheme. Finally, simulation results will be presented in order to verify applicability and the effectiveness of the proposed learning control.

Problem formulation

Biomedical robotics has attracted many researchers who developed and significantly improved different treatments and medical procedures. Particularly, the concept of robotics assisted therapy is one of the most important issues in social robotic research where various robotic devices are proposed to help stroke patients and physiotherapists during the rehabilitation training, [11]. However, the design of the control system remain one of the main difficulties specially when intending to realize predefined complex movements and recovering at the same time motion and force human capabilities. In addition, in paper [10] it is shown that fractional models are more adequate to describe human arm behaviour; the are simpler, more exact and with less parameter uncertainty. One of the reasons is due to fact that muscles show viscoelastic behaviour, and they can also be modelled using fractional derivatives. Consequently,mathematical models for the described class of complex systems contain fractional order differential equations with an associate algebraic equation, which outlines constrained system dynamics. Here,such a system is considered to be a fractional order nonlinear singular system described by the pseudo-state space and output equations:

$$ED^{\alpha}\, \boldsymbol{x}_{i+1}(t) = f_{i+1}(t, x_{i+1}) + B(t)u_{i+1}(t), \quad 0 < \alpha < 1 \tag{1}$$

$$\boldsymbol{y}_{i+1}(t) = C(t)\boldsymbol{x}_{i+1}(t), \tag{2}$$

Here, the left Caputo fractional order operator is used, $0 \le \alpha < 1$:

$${}_{t_0}^{C}D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} (t-\tau)^{-\alpha} \frac{df(\tau)}{d\tau} d\tau. \tag{3}$$

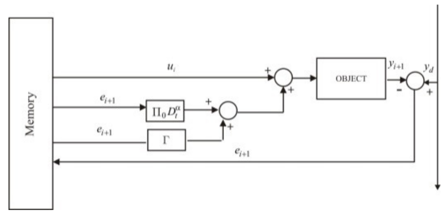

Figure 1. Block diagram of closed-loop PD^{α} iterative learning control for a nonlinear singular system

In this work, beside ILC of integer order [11], fractional order ILC control of a complex dynamic model of the arm-support system as nonlinear singular system is proposed as follows, (Fig.1):

$$u_{i+1}(t) = u_i(t) + \Gamma(t) \cdot e_{i+1}(t) + \Pi(t) \cdot {}_0D_t^{\alpha} e_{i+1}(t), \tag{4}$$

where $\Gamma(t), \Pi(t)$ are gain matrices of appropriate dimensions, i -denotes ith-iteration and t is the time within the operation interval $J = [t_o, t_o + T], J \subset R$. A sufficient condition for convergence of a proposed feedback ILC will be given and proved. Proposed feedback PD^{α} type of fractional order ILC will be examined for changes of gain parameters of proposed ILC scheme.

Conclusions

This paper studies the problem of control a complex dynamic model of the arm-support system. This complex system can be modeled as nonlinear singular system of fractional order. To the best of our knowledge for the first time, one fractional order feedback PDalpha type of ILC scheme is proposed to solve the trajectory tracking problem of nonlinear singular system of fractional order. The sufficient conditions for the convergence in time domain of a proposed ILC were given by the corresponding theorem and proved.

Aknowledgments

This research was supported by the research grants of the Serbian Ministry of Education, Science and Technological Development under the numbers OI 174001, TI 35006,III 41006 and partially supported by Serbia-China bilateral project under the number 3-12.

References

[1] Xu J.X., Tan Y. (2003) Linear and Nonlinear Iterative Learning Control. Springer-Verlag Berlin Heidelberg.

[2] Xu J.X., Panda S.K., Lee T.H, (2009) Real-time Iterative Learning Control Design and Applications. Springer-Verlag London Lim.

[3] Buzurović I., (2007) Dynamic model of medical robot represented as descriptor system, *International journal of information and systems sciences*, 2(2): 316-333.

[4] Dai L. (1989) Singular Control Systems,118 Lecture Notes in Control and Information Sciences. New York: Springer-Verlag.

[5] Monje C.A., Chen,Y.Q, Vinagre B.M., Xue D.Y., Feliu-Batlle V.(2010) Fractional-order Systems and Controls: Fundamentals and Applications. London: Springer-Verlag.

[6] Kaczorek T., (2013) Singular fractional continuous-time and discrete-time linear systems. *Acta mechanica et automatica* (7)1: 26-33.

[7] Lazarević M.P., (2003),PDalpha-type iterative learning control for fractional LTI system, *in Proc.4th Int. Carpath. Control Conf., High Tatras,* Tatr.Lomnica, Slovak Republic, 869–872.

[8] Yan L.,Wei J., (2015) Fractional order nonlinear systems with delay in iterative learning control, *Applied Math. and Com.* 257, 546–552.

[9] Lazarević, M. P., Tzekis, P. (2016). Robust second-order PD type iterative learning control for a class of uncertain fractional order singular systems. *Journal of Vibration and Control*, 22(8): 2004–2018.

[10] Ventura A., Tejado I., Valerio D., Martins J., (2015) Fractional direct and inverse models of the dynamics of a human arm. *Journal of Vibration and Control*,1–15, DOI: 10.1177/1077546315580471

[11] .Freeman C.F.,Rogers E.,Burridge J.H., Hughes Meadmore A.K., (2015) Iterative Learning Control for Electrical Stimulation and Stroke Rehabilitation, Springer-Verlag London Ltd.