Non-linear dynamics of a heavy mass particle and rolling ball along curvilinear trace of series of circle arcs: Phase trajectory portraits, some analogies and vibro-impacts

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Summary. Constructions of the phase trajectory portraits of a heavy mass particle and rolling ball along curvilinear trace containing series of circle arcs are presented. An analysis of structural properties of phase trajectory portraits is realized. Conditions of the existence of a trigger of coupled three singular points in phase portraits and homoclinic orbit in the form of number "eight" are defined. An analogy between phase portraits of a heavy mass particle motion along curvilinear trace of series of circle arcs, rolling disk and rolling ball along same curvilinear trace is identified. Using these phase trajectory portraits and extended classical theory of impacts with kinematics and dynamics of rolling bodies the methodology of vibro-impact system dynamics is formulated. In the phase portraits, series of energy jumps between rolling bodies in collision are observed.

Principal models

In Figure 1.**a** a model of a moving heavy mass particle along curvilinear trace consisting of series of circle arcs in vertical stationary plane is presented. Ordinary differential equations describing dynamics of the heavy mass particle movements along corresponding circle arc are in the form [1, 4, 5]:

$$\ddot{\varphi}_i + \frac{g}{R}\sin\varphi_i = 0, \quad i = 1,3 \quad \text{and} \quad \ddot{\varphi}_2 + \frac{g}{R_0}\sin(\varphi_2 + \pi) = 0 \tag{1}$$

Corresponding equations of branches of phase trajectories are in the following forms:

$$\dot{\varphi}_i^2 - \dot{\varphi}_{i,0}^2 + \frac{2g}{R}\{[1-\cos\varphi_i] - [1-\cos\varphi_{i,0}]\} = 0, \quad i = 1,3 \ \text{and} \ \dot{\varphi}_2^2 - \dot{\varphi}_{2,0}^2 + \frac{2g}{R_0}\{[1-\cos(\varphi_2+\pi)] - [1-\cos(\varphi_{2,0}+\pi)]\} = 0 \tag{2}$$

In Figure 1.**b** Model of a rolling heavy ball (or disk) along curvilinear trace consisting of series of circle arcs in vertical stationary plane is presented. Ordinary differential equations describing dynamics of this rolling body (disk or ball) along corresponding circle arc are in the form:

$$\ddot{\varphi}_i + \frac{g}{\kappa_i(R-r)}\sin\varphi_i = 0, i = 1,3, \ \text{and} \ \ddot{\varphi}_2 + \frac{g}{\kappa_2(R_0+r)}\sin(\varphi_2+\pi) = 0, \text{where} \ \kappa_i = \frac{\mathbf{J}_{Ci}}{Mr^2} + 1 = \frac{i_{Ci}^2}{r^2} + 1 \tag{3}$$

where κ_i, $i = 1,2,3$ coefficient of rolling disk (ball) and $\mathbf{J}_{Ci}$ is axial mass inertia moment for instantaneous axis of rolling.

Corresponding equations of branches of phase trajectories are in the following forms:

$$\dot{\varphi}_i^2 = \dot{\varphi}_{i,0}^2 + \frac{2g}{\kappa_i(R-r)}(\cos\varphi_i - \cos\varphi_{i,0}), \quad i = 1,3, \ \text{and} \ \dot{\varphi}_2^2 = \dot{\varphi}_{2,0}^2 + \frac{2g}{\kappa_2(R_0+r)}(\cos\varphi_2 - \cos\varphi_{2,0}) \tag{4}$$

There are Mathematical and qualitative analogy between dynamics of these mechanical models.

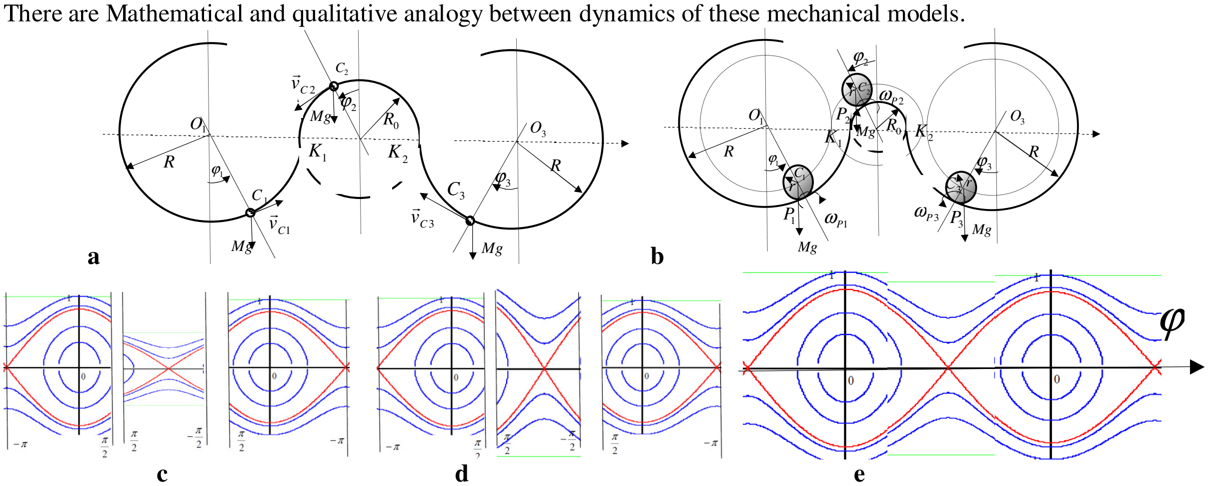

Figure 1. a Model of a moving heavy mass particle along curvilinear trace consisting of series of circle arcs in vertical stationary plane; **b** Model of a rolling heavy ball (or disk) along curvilinear trace consisting of series of circle arcs in vertical stationary plane; **c , d** and **e** Different compositions of phase trajectory portraits of different phase trajectory branches in phase portraits and curves of constant total mechanical energy branches of two rolling disks for motion in intervals along different arcs of curvilinear traces composed by three coupled circle arcs with different radii.

Analysis of structural properties of analogous phase trajectory portraits

Using equations (2) or/and (4) corresponding segments of the phase portraits for different geometric parameters and kinetic initial conditions (initial velocity and initial angular position on the curvilinear trace) and by computer software tools are drown and presented in Figure **c, d** and **e.** Different compositions of the phase trajectory portraits of different phase trajectory branches in phase portraits and curves of constant total mechanical energy branches of two rolling disks for motion in intervals along different arcs of curvilinear traces composed by three coupled circle arcs with different radii are presented. We can see that maximum of the curvilinear trace caused an unstable singular point saddle type that corresponds to unstable equilibrium position with local maximum of rolling body potential energy. Depending of geometric and kinetic parameters in the phase portraits exists a trigger of coupled three singular points [2] around which exists a homclinic closed phase trajectory in the form of number “eight”.

Methodology for investigation of a class of the vibro-impact dynamics with rolling bodies collisions

In Reference [3] are presented some dynamical parameters of two rolling bodies in collision like: a new hypothesis of angular momentum conservation, a new definition of the coefficient of restitution of collision defined by angular velocity. Parameters are defined for the cases before and after collision. Also, new expressions of outgoing angular velocities after collision of two rolling bodies are derived. These results are extension of classical theory of collision with kinematics and dynamics of collision of the rolling bodies. Using these results and classical method of phase trajectory portraits in phase plane and portrait of the constant total mechanical energy curves we founded an effective methodology for investigation of a class of the vibro-impact dynamics in the systems with collision of the rolling bodies. Energy jumps between bodies in collision and characteristic discontinuity appear in the collisions [6]. Methodology for investigation of a class of the vibro-impact dynamics with rolling bodies collisions founded on the phase plane method and extended classical theory of collision, theoretically is clear, but for solving particular tasks appear difficulties in determination of the position of each of successive collisions and, also time of each collision, arrival angular velocities. Consequently, it is necessary to include numerical method for solving elliptic integrals, roots of nonlinear transient equations. Energy analysis, using phase trajectory portraits and portrait of constant mechanical energy curves, give possibility to indicate jumps in directions of velocity before and after collisions and kinetic energy jumps between rolling bodies in collisions. Particular cases are in the case that representative points of nonlinear dynamics of each rolling body in collision have jumps from one to other phase trajectory in the form of number „eight.

Conclusions and perspectives

In the case that previous models in Figure 1. a and 1.b rotate around vertical axis of symmetry, in phase trajectory portrait appearance of numbers of new triggers of coupled singular points is possible [2] in each parts of phase trajectory portraits corresponding to each of the circle arcs of the rotate models. Multiplication of the triggers of the coupled singular points and appearance of a number of homoclinic trajectories in the form of number „eight” are caused by coupled rotations.

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References

[1] Hedrih (Stevanović) R. K. (2005) Nonlinear Dynamics of a Heavy Material Particle Along Circle which Rotates and Optimal Control. *Chaotic Dynamics and Control of Systems and Processes in Mechanics* (Eds: G. Rega, and F. Vestroni), p. 37-45. IUTAM Book, in Series *Solid Mechanics and Its Applications, Editerd by G.M.L. Gladwell, Springer.* 2005, XXVI, 504 p

[2] Hedrih (Stevanović) R. K. (2004). A Trigger of Coupled Singularities, *MECCANICA* 39(3):,. 295-314. DOI: 10.1023/B:MECC.0000022994.81090.5f,

[3] KaticaR. (Stevanović) Hedrih,(in press) Dynamics of Impacts and Collisions of the Rolling Balls. *Dynamical Systems: Theoretical and Experimental Analysis. Springer Proceedings in Mathematics & Statistics*, 182, Ch13:157-168.Doi:10.1007/978-3-319-42408-8

[4] Hedrih (Stevanović) R. K.,(2000) Nonlinear Dynamics of a Gyro-rotor, and Sensitive Dependence on initial Conditions of a Heav Gyro-rotor Forced Vibration/Rotation Motion, Semi-Plenary Invited Lecture, *Proceedings: COC* 2000, Edited by F.L. Chernousko and A.I. Fradkov, IEEE, CSS, IUTAM, SPICS, St. Petersburg, Inst. for Problems of Mech. Eng. of RAS,. 2: . 259-266.

[5] Hedrih (Stevanović) K. (2009) Vibrations of a Heavy Mass Particle Moving along a Rough Line with Friction of Coulomb Type, *International Journal of Nonlinear Sciences & Numerical Simulation* 10(11):1705-1712.http://www.freundpublishing.com/International_Journal_Nonlinear_Sciences_Numerical%20Simulation/MathPrev.htm**.**

[6] Hedrih (Stevanović) K. (2011) Discontinuity of kinetic parameter properties in nonlinear dynamics of mechanical systems, **Keynote Invited Lecture**, 9th Brazilian Conference on Dynamics, Control and Their Applications - 9º Congresso Temático de Dinâmica, Controle e Aplicaçõesm, June 07-11, 2010. UneSP, Sao Paolo (Serra Negra), Brazil, *Post conference Proceedings of the 9th Brazilian Conference on Dynamics Control and their Applications, Serra Negra, 2010*:. 8-40. , http://www.rc.unesp.br/igce/demac/dincon2010/instrucao.php