Implicit finite element formulation of the inverse dynamics of vibrating robots

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Summary. Inverse dynamics methods are useful for the design feedforward control of flexible robots. In this work, a general finite element framework is proposed for the formulation of the inverse dynamics problem, which can account for the flexibility in the links and joints and for arbitrary mechanical topologies, e.g., including closed kinematic loops. The dynamics between the joint torques and the tip motion of a flexible manipulator is generally not minimum-phase, however, practical inverse dynamics solutions can be obtained based on the formulation of a two-point boundary value problem or an optimization problem. The proposed methodology is illustrated for the feedforward control of a parallel kinematic robot with three flexible links.

Introduction

Today, the interest for lightweight robots is growing, especially for the design of cooperative robots able to interact safely with a human operator. Firstly, the motion of a lightweight robot involves less kinetic energy so that the robot can brake faster if a risk of collision is present and the transferred energy is reduced if an impact cannot be avoided. However, the reduction in mass is still limited by the usual design constraint that the eigenfrequency of vibration should be two or three time above the motion bandwidth, which is a necessary assumption to neglect the flexibility of the arm in the control design procedure. If this constraint is disregarded, significant mass reductions can be expected but the challenge is then to fully account for the participation of the vibrating modes to the dynamic response in the control design procedure.
Hence, the present work focuses on the design of controllers for vibrating robots with a specific focus on feedforward controllers, which aim at planning the control actions based on a anticipation of the system dynamics. In practice, such feedforward control scheme should be combined with a feedback control action in order to guarantee robust performances. The planning procedure usually relies on the dynamic inversion of the system model, which maps a desired trajectory into some appropriate control inputs.
In some special cases, the inverse dynamics can be evaluated based on some assumptions on the system forward dynamics, such as linearity, no internal dynamics, stable internal dynamics, planar motion, absence of bilateral constraint and/or absence unilateral constraint. But none of these assumptions are met in the general case where large 3D motions, configuration changes, link and joint flexibility, closed kinematic loops and contact conditions can be expected. Therefore, the formulation of the problem should be well-posed in an appropriate mathematical framework.

Proposed methodology

The finite element approach is selected here for the formulation of the inverse dynamics problem. Indeed, this approach appears as a general modelling framework for flexible mechanical systems. More precisely, it can naturally represent systems with arbitrary mechanical topology composed of rigid and flexible bodies interconnected by kinematic joints. Regarding the spatial discretization process, the flexible links can be represented either by a lineic mesh of beam elements, a 3D mesh of volume elements or a reduced-order model which is restricted to a few low-frequency vibration modes. This third technique is also known as the superelement approach in the finite element world.
After this spatial discretization, the configuration of the system is represented by 3D translation and rotation variables collected in the variable $\mathbf{q}$ which belong to an n-dimensional manifold G. Due to the presence of the kinematic joints, the variable $\mathbf{q}$ does not evolve freely on the space G but have to satisfy m constraints $\mathbf{\Phi}(\mathbf{q}) = \mathbf{0}$. The desired trajectory is then included in the problem as an additional set of s time-dependent constraints of the general form $\mathbf{h}(\mathbf{q}) = \mathbf{y}_d(t)$, where $\mathbf{y}_d(t)$ represents the desired output trajectory. Thus, the inverse problem is formulated as a differential-algebraic equation (DAE)

$$\dot{\mathbf{q}} = \mathbf{G}\mathbf{v} \tag{1a}$$

$$\mathbf{M}(\mathbf{q})\dot{\mathbf{v}} + \mathbf{g}_f(\mathbf{q},\mathbf{v}) + \mathbf{\Phi}_{\mathbf{q}}^T(\mathbf{q})\,\boldsymbol{\lambda} = \mathbf{A}(\mathbf{q})\mathbf{u} \tag{1b}$$

$$\mathbf{\Phi}(\mathbf{q}) = \mathbf{0} \tag{1c}$$

$$\mathbf{h}(\mathbf{q}) = \mathbf{y}_d(t) \tag{1d}$$

where the differential equations includes the velocity compatibility condition (1a) and the dynamic equilibrium (1b), whereas the algebraic equations includes the kinematic constraints (1c) and the prescribed trajectory or servo-constraint (1d). In this expression, $\mathbf{v}$ is the n-dimensional velocity vector, $\mathbf{G}$ is velocity compatibility matrix, $\boldsymbol{\lambda}$ is the m-dimensional vector Lagrange multiplier vector, $\mathbf{u}$ is the s-dimensional vector . This DAE turns out to be an implicit representation of the internal dynamics of the system, known in nonlinear control theory, which is defined as the remaining dynamics of the system when the output trajectory is prescribed and when the control inputs are left free.
Based on this general but implicit formulation, the properties of the internal dynamics are investigated and the question of the initial conditions is addressed. In practice, solutions based on fixed initial conditions, which can be obtained by

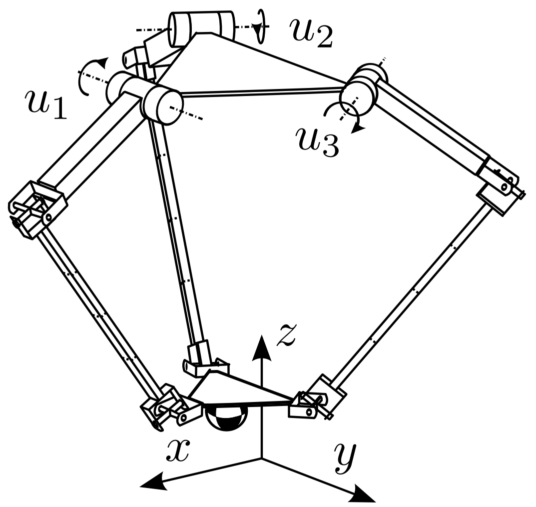

Figure 1: Parallel kinematic robot with flexible links

a simple forward integration of Eq. (1), are limited to systems with a stable internal dynamics (i.e., minimum-phase systems). Vibrating robot with non-collocated inputs and outputs often exhibit an unstable internal dynamics so that solutions based on fixed initial conditions are unbounded and of little practical interest. Instead, relaxed initial conditions have to be considered in this case, e.g., formulations based on a two-point boundary value problem [3, 2] or on an optimization problem [1].
This implicit formulation of the internal dynamics is convenient from a user perspective as (i) it can be automatically obtained from a high-level user interface (ii) it can be solved using robust numerical algorithms for DAEs. The method and its generality is illustrated using the example of a 3D parallel kinematic robot with three flexible links illustrated in Figure 1.

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