Bifurcations of relative equilibria sets of a massive point on an uniformly rotating spherical asteroid

Alexander A. Burov* **, Ivan I. Kosenko *** **** , and Ekaterina S. Shalimova*****
*Dorodnicyn Computing center of the Federal Research Center "Computer science and Control", Russian Academy of Sciences, Moscow, Russia
** National research university "Higher School of Economics", Moscow, Russia
*** Department of Mechanics and Mecatronics, Faculty of Mechanics and Mathematics, Lomonosov Moscow State University, Moscow, Russia
**** Department of Theoretical Mechanics, Moscow Aviation Institute, Moscow, Russia
*****Institute of Mechanics, Lomonosov Moscow State University, Moscow, Russia

Summary. The motion of a point particle on a surface of a spherical asteroid under the action of dry friction force is studied. It is assumed that the asteroid rotates with a constant angular velocity about a vertical axis passing through its center. Sets of the non-isolated equilibrium positions are described, and their dependence on parameters of the system is investigated. The results of the investigation are represented in the form of bifurcation diagrams.

Introduction

According to the latest investigations ([2, 1]), for small celestial bodies the centrifugal forces of their rotation exceed their own gravitational fields. This leads to the fact, that unfixed objects on their surfaces cannot be located in certain areas of the surfaces. Another effect, namely "sliding" of the objects from certain areas of the surfaces of rotating celestial body can occur, because the friction force is too small to hold the objects. In this paper such areas and their dependence on parameters of the system are studied.
The fact of existence of these sets for the systems with dry friction is well-known ([8]). The systematical investigation of dependence of these sets began with papers of A.P. Ivanov [5]. Earlier bifurcations of such sets were studied for some particular problems ([7, 3]).
The stability theory for systems with dry friction were developed by G.K.Pozharitsky [9]. Methods of investigation of stability of equilibria of this kind based on the general theory of systems with discontinuous right-hand side were later developed ([4, 6]).
In this paper the system with two degrees of freedom with one bifurcation parameter is studied. The results of the investigation are represented in the form of bifurcation diagrams. Another example of a system with two degrees of freedom and one bifurcation parameter that arises in the study of self-balancing system was considered earlier in [10].

Statement of the problem

The motion of a heavy point P on a surface of a spherical celestial body is considered. Let m be a mass of the point and O be a center of gravity of the body. Let $Oxyz$ be a relative coordinate system, Ox, Oy, Oz are principal central axes of inertia of the body (Fig.1). It is assumed that point moves under action of Newtonian gravitational attraction force and dry friction force with coefficient of friction μ.

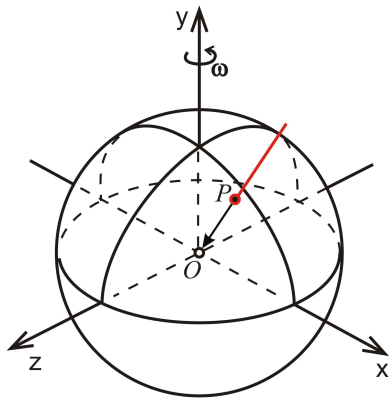

Figure 1: A point on a spherical asteroid

Let the position of the point be given by $\mathbf{r} = (x, y, z)$. Then the constraints restricting its motion are defined by the relation

$$f = \frac{1}{2}\left(x^2 + y^2 + z^2 - \ell^2\right) = 0 \tag{1}$$

Using dimensionless coordinates and parameters, the equations of motion of the system can be written as

$$
\begin{gathered}
\lambda=-(\dot{x}^2+\dot{y}^2+\dot{z}^2)-\omega^2(x^2+z^2)+2\omega(\dot{z}x-\dot{x}z)+1,\\
\ddot{x}=\omega^2x-2\omega\dot{z}+\lambda x-x+F_x,\quad \ddot{y}=\lambda y-y+F_y,\\
\ddot{z}=\omega^2z+2\omega\dot{x}+\lambda z-z+F_z.
\end{gathered} \tag{2}
$$

where λ is Lagrange's multiplier.
The condition of relative equilibrium of the point is

$$
\xi(x,y,z)\le\mu^2\eta^2(x,y,z),\quad x^2+y^2+z^2=1, \tag{3}
$$

where

$$
\begin{gathered}
\xi(x,y,z)=(((x^2+z^2)x-x)^2+(x^2+z^2)^2y^2+((x^2+z^2)z-z)^2)\omega^4=\\
\eta(x,y,z)=1-\omega^2(x^2+z^2).
\end{gathered}
$$

On Fig.2 the obtained equilibrium sets are represented on a plane (θ,ω), where θ is an angle of inclination of the vector OP to the vertical. The sets are denoted by grey color.

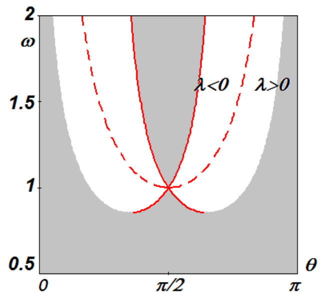

Figure 2: Bifurcation diagram for $\mu=0.7$

Lyapunov stability of the points of the equilibrium sets is also discussed. The stability of the interior points was proved in [9]. In order to study the stability of boundary points let us now introduce a function

$$
C=\left(\frac{\partial f}{\partial r},F_c+F_N+N\right)_{f=0}, \tag{4}
$$

where F_c is a centrifugal force, F_N is a gravity force, N is a normal reaction force, $f(x,y,z)=\xi(x,y,z)-\mu^2\eta^2(x,y,z)$. This function defines the so-called 'statical' stability of the boundary points. Let us put a point P on a boundary point Q and then 'release' it with zero initial velocity. Assume now, that there is no friction in a system, so $\mu=0$. If $C<0$ in point Q then the point P will begin to move into the set or along ins boundary. If $C>0$ then the point P will begin to move away from the set, so the point Q is unstable.
Let us say that the component of equilibrium set is attracting if all points on its boundary are 'statically' stable. On fig.2 unstable parts of the boundaries are denoted by red color.
Thus if the angular velocity is small, the whole asteroid is an equilibrium set. If $\omega=\omega_\star$ this set divides on three components, one of them is near the equator and two are near the poles. The areas near the poles are getting smaller with the increase in the angular velocity, and these components remain attracting. The area near the equator gets smaller when $\omega\mapsto 1$, and its boundary points are 'statically' unstable. If $\omega>1$ there are no equilibria near the equator, for which $\lambda>0$.

References

[1] Yu Jiang, Yun Zhang, Hexi Baoyin, Surface motion relative to the irregular celestial bodies // Planetary and Space Science. 2016. Vol.127, P.33–43.

[2] Bellerose J., Girard A., Scheeres DJ Dynamics and Control of Surface Exploration. Robots on Asteroids // In: Optimization & Cooperative Control Strategies. LNCIS 381. M.J. Hirsch et al. (Eds.). Berlin, Heidelberg: Springer-Verlag. 2009. P.135 – 150.

[3] Biemond J.J.B. Nonsmooth dynamical systems. On stability of hybrid trajectories and bifurcations of discontinuous systems // PhD Thesis. Eindhoven. 2012. 217 p.

[4] Ivanov A. P., The stability of equilibrium in systems with friction // J. Appl. Math. Mech., 2007. Vol. 71. No. 3. P. 385-395;

[5] Ivanov A. P.,Fundamentals of the Theory of Systems with Friction. Moscow-Izhevsk: "Regular and chaotic motion", Institute of Computer Science, 2011.

[6] Leine R.I., van de Wouw N. Stability properties of equilibrium sets of nonlinear mechanical systems with dry friction and impact // Nonlinear Dynamics. 2008. V.51. No. 4. P. 551–583.

[7] Leine R.I., van Campen D.H. Bifurcation phenomena in non-smooth dynamical systems // Europ. J. Mechanics A. Solids. 2006. V.25. P.595 – 616.

[8] Magnus K. Schwingungen. Eine Einführung in die theoretische Behandlung von Schwingungsproblemen. - Stuttgart: Teubner, 1. Aufl. 1961.

[9] Pozharitsky, G.K. Stability of equilibria for the systems with dry friction // PMM, 1962. Vol. 26, No. 1, P. 5–14.

[10] van de Wouw N., van den Heuvel M.N., van Rooij J.A., Nijmeijer H. Performance of an automatic ball balancer with dry friction // Int. J. Bifurcation and Chaos. 2005. Vol.15. P.65 – 82.