Plane motion of the rigid body with the spring-damper suspension

Roman Starosta*, Grażyna Sypniewska-Kamińska * and Jan Awrejcewicz**
* *Poznan University of Technology, Institute of Applied Mechanics, Poznań, Poland*
** *Łódź University of Technology, Department of Automatics, Biomechanics and Mechatronics, Łódź, Poland*

Summary. The paper deals with the analytical investigation of the behavior of the harmonically excited physical pendulum suspended on the nonlinear spring. The asymptotic method of multiple scales (MS) has been adopted in order to carry out the analytical computations. The solutions to the equations of motion up to the third order have been achieved. Such an approach allows one to perform a qualitative analysis of the behavior of the system. MS method allows one, among others, to recognize all possible resonance conditions which can appear in the system.

Introduction

Although pendulums are relatively simple systems, they can be used to simulate the dynamics of a variety of engineering devices and machine parts. The behavior of pendulum-type mechanical systems with nonlinear and parametric interactions is complicated, and hence its understanding and prediction are important from a point of view of both the theory and application. The coupling of the equations of motion results in a possibility of autoparametric excitation and is connected to the energy exchange between vibration modes [4]. Various kinds of pendulums are widely discussed in numerous references and analytical investigations are recently of great interest of many researchers. In the paper [1], the kinematically driven spring mathematical pendulum is tested near main and parametric resonances. The 3 degree-of-freedom (DOF) system with a double pendulum is analytically investigated in the paper [3]. The physical pendulum suspended on the a spring has been modelled and discussed in the article [2]. The present paper extends these investigations and presents the results of further development of the model.

Formulation of the problem

The mass-spring-damper system studied in the paper and presented in Figure 1 is constrained to the planar motion. The mass of the rigid body suspended at the point O is equal m. I_A denotes the body moment of inertia about the axis which passes through the point A and is perpendicular to the plane of motion. The mass center of the body is located at the point C, therefore the eccentricity $E=AC$. The spring is assumed to be massless and nonlinear of cubic type, and k_1 and k_2 are constant coefficients. The extension Z of the spring and angles φ and ψ are used as the general coordinates. The system is driven by the harmonically changing torques $M_\varphi(t) = M_2\cos(\Omega_2 t)$ and $M_\psi(t) = M_3\cos(\Omega_3 t)$, and the force $F_1(t) = F_1\cos(\Omega_1 t)$ which acts at the point A along the spring. A linear damper with constant C_1 is included in the system. Furthermore, two torques of viscous nature attenuate the swing vibration related to the angles φ and ψ (C_2 and C_3 are viscous coefficients).

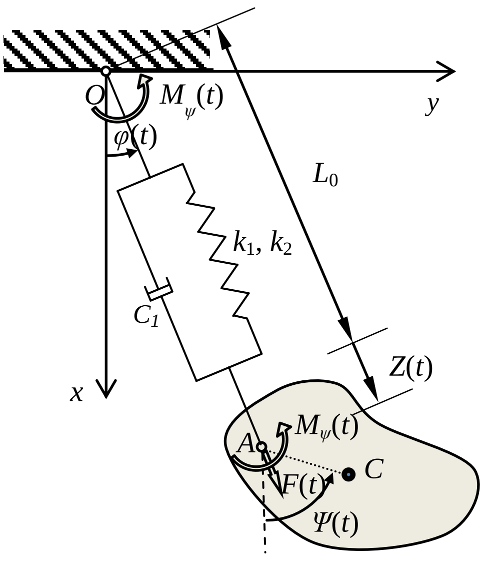

Figure 1. Mass-spring-damper system

The dimensionless forms of equations of the system motion are as follows

$$\ddot{z}+c_1\dot{z}+z+\alpha z^3+3z_r\alpha z^2+3z_r^2\alpha z-w_2^2(\cos\varphi-1)-(1+z)\dot{\varphi}^2-e\cos(\varphi-\psi)\dot{\psi}^2+e\sin(\varphi-\psi)\ddot{\psi}=f_0\cos(p_1\tau), \quad (1)$$

$$\ddot{\varphi}(1+2z+z^2)+w_2^2\sin\varphi(1+z)+c_2\dot{\varphi}+2\dot{z}\dot{\varphi}+2z\dot{z}\dot{\varphi}+e\sin(\varphi-\psi)(1+z)\dot{\psi}^2+e\cos(\varphi-\psi)(1+z)\ddot{\psi}=m_{01}\cos(p_2\tau), \quad (2)$$

$$\ddot{\psi}+w_3^2\sin\psi+c_3\dot{\psi}+2\frac{w_3^2}{w_2^2}\cos(\varphi-\psi)\dot{z}\dot{\varphi}-\frac{w_3^2}{w_2^2}(1+z)\sin(\varphi-\psi)\dot{\varphi}^2+\frac{w_3^2}{w_2^2}\sin(\varphi-\psi)\ddot{z}+\frac{w_3^2}{w_2^2}\cos(\varphi-\psi)\ddot{\varphi}=m_{02}\cos(p_3\tau), \quad (3)$$

where z, φ, ψ are generalized coordinates and functions of the dimensionless time $\tau=\omega_1 t$.
The dimensionless quantities are defined as follows: $e=E/L$, $z=Z/L$, $L=L_0+Z_r$ where L_0 is the length of the non-stretched spring, Z_r is the spring static elongation, and $z_r=Z_r/L$ is its dimensionless counterpart which fulfils the equation

$$\alpha z_r^3+z_r=w_2^2. \quad (4)$$

Assuming $\omega_1=\sqrt{\frac{k}{m}}$ as the reference quantity, we introduce the dimensionless parameters as follows

$c_1=\frac{C_1}{m\omega_1}$, $c_2=\frac{C_2}{mL^2\omega_1}$, $c_3=\frac{C_3}{\omega_1 I_A}$, $f_0=\frac{F_0}{mL\omega_1^2}$, $m_{01}=\frac{M_1}{mL^2\omega_1^2}$, $m_{02}=\frac{M_2}{\omega_1^2 I_A}$, $w_2=\frac{\omega_2}{\omega_1}$, $w_3=\frac{\omega_3}{\omega_1}$, $p_1=\frac{\Omega_1}{\omega_1}$, $p_2=\frac{\Omega_2}{\omega_1}$, $p_3=\frac{\Omega_3}{\omega_1}$, where $\omega_2^2=\frac{g}{L}$ and $\omega_3^2=\frac{Egm}{I_A}$.

Equations (1)–(3) should be supplemented by the initial conditions for generalized coordinates and their first derivatives

$$z(0)=u_{01},\ \dot{z}(0)=u_{02},\ \varphi(0)=u_{03},\ \dot{\varphi}(0)=u_{04},\ \psi(0)=u_{05},\ \dot{\psi}(0)=u_{06}. \quad (5)$$

The trigonometric functions of arguments φ and ψ are approximated by the power series up to the third order in a neighborhood of the static equilibrium position. The MS method is applied to solve the governing Eqs. (1)–(3).
The functions z, φ, and ψ are approximated by the power series of the small perturbation parameter ε and can be presented in the form

$$z=\sum_{k=1}^{3}\varepsilon^k x_k(\tau_0,\tau_1,\tau_2)+O(\varepsilon^4),\ \varphi=\sum_{k=1}^{3}\varepsilon^k\phi_k(\tau_0,\tau_1,\tau_2)+O(\varepsilon^4),\ \psi=\sum_{k=1}^{3}\varepsilon^k\zeta_k(\tau_0,\tau_1,\tau_2)+O(\varepsilon^4) \quad (6)$$

where $\tau_0=\tau,\ \tau_1=\varepsilon\tau,\ \tau_2=\varepsilon^2\tau,$ are various time scales.
The approximate analytical solution obtained using the MS method allows one to detect all possible resonance cases. The appropriate approach gives the possibility to determine the frequency response of the system in resonance and also to estimate stability of the obtained solutions.

Conclusions

The approximate solution to the governing equations, up to the third order, has been obtained using the MS method with three time scales. The analytical form of this solution is the main advantage of the applied approach, giving the possibility of the qualitative and quantitative study of the system dynamics in a wide range of the frequency spectrum. The adequate conditions for all possible resonances have been derived. The applied approach allows for studying up to three resonances occurring simultaneously. In addition, the proposed procedure gives a possibility to discuss both steady and non-steady vibrations of our forced system.

Acknowledgments
This paper was financially supported by the grant of the Ministry of Science and Higher Education in Poland realized in Institute of Applied Mechanics of Poznan University of Technology (DS-PB: 02/21/DSPB/3477).

References

[1] Awrejcewicz J., Starosta R., Sypniewska-Kamińska G. (2011) Resonances in kinematically driven nonlinear spring pendulum. *DSTA 11th Conference*, 103-108.
[2] Awrejcewicz J., Starosta R., Sypniewska-Kamińska G. (2013) Asymptotic analysis of resonances in nonlinear vibrations of the 3-dof pendulum. *Differ. Equ. Dyn. Syst.* **21**(1&2):123-140.
[3] Sado D., Gajos K. (2008) Analysis of vibrations of three-degree-of-freedom dynamical system with double pendulum. *J. Theor. App. Mech.,* **46**(1):141-156.
[4] Starosvetsky, Y. & Gendelman O.V. (2008) Dynamics of a Strongly Nonlinear Vibration Absorber Coupled to a Harmonically Excited Two-Degree-of-Freedom System, *J. Sound Vib.* **312**:234-256.