Passive realization of a nonlinear piezoelectric tuned vibration absorber with a saturable inductor

Boris Lossouarn[1,2], Jean-François Deü[1] and Gaetan Kerschen[2]
[1]*LMSSC, Conservatoire national des arts et métiers, Paris, France*
[2]*Department of Aerospace and Mechanical Engineering, University of Liège, Liège, Belgium*

Summary. The objective of this study is to develop a fully passive nonlinear piezoelectric vibration absorber that mitigates a specific resonance of a nonlinear system. To this end, the saturation of a passive inductor based on a magnetic circuit in ferrite material is exploited to realize the nonlinearity in the absorber. The resulting variable inductance generates an autonomous adjustment of the shunt resonance frequency. The performance of the proposed device is demonstrated numerically and experimentally.

Introduction

Tuned vibration absorbers can effectively mitigate specific resonances of mechanical systems [1, 2, 3]. Yet, mechanical nonlinearities introduce a mistuning that significantly reduces the damping performance. In [4, 5], Habib et al. developed a nonlinear vibration absorber, the nonlinearity of which possesses the same mathematical form as that of the primary system. This principle of similarity was extended to piezoelectric tuned vibration absorbers using nonlinear electrical shunts [6]. Based on those results, the present study aims at designing a passive resonant shunt that synthesizes a cubic nonlinearity. Instead of using a nonlinear capacitor, a physical inductor subjected to magnetic saturation is considered. An increase of the electrical current flowing through the shunt leads to a decrease of the equivalent inductance, which generates a hardening nonlinearity in the electrical domain. Through an adequate magnetic circuit, it becomes possible to apply the principle of similarity with purely passive electrical components.
The next section shows the limits of a linear piezoelectric tuned vibration absorber for vibration mitigation of nonlinear structures. It introduces the concept of a nonlinear piezoelectric tuned vibration absorber (NPTVA), which relies on a principle of similarity with the host structure. We then show how a fully passive NPTVA can be realized using a saturable inductor.
For demonstration, a beam possessing a cubic nonlinearity and covered with piezoelectric patches is considered. A linear shunt first validates the setup at low forcing amplitudes. For higher amplitudes, a nonlinear inductor is designed and incorporated into the piezoelectric shunt. Finally, experimental results compare the performance of both linear and nonlinear piezoelectric tuned vibration absorbers.

The nonlinear piezoelectric tuned vibration absorber (NPTVA)

Limits of the linear resonant shunt

A piezoelectric tuned vibration absorber consists of a piezoelectric element shunted with an inductor and a resistor in order to mitigate mechanical vibrations [1, 2, 3]. The shunt generates an electrical resonance that is tuned to the target mechanical resonance frequency. Considering a single-degree-of-freedom mechanical model, the resulting coupled system of equations can be written as follows,

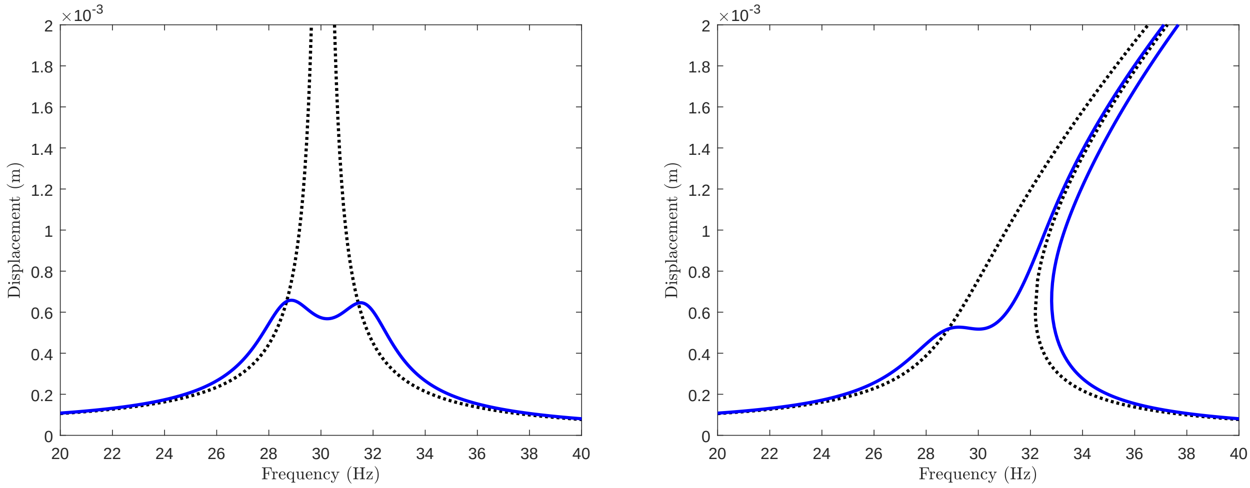
$$\begin{aligned} m\ddot{u} + \left(K^E + \frac{e^2}{C^\varepsilon}\right) u &= f + \frac{e^2}{C^\varepsilon}\frac{q}{e} \\ e^2 L \frac{\ddot{q}}{e} + e^2 R \frac{\dot{q}}{e} + \frac{e^2}{C^\varepsilon}\frac{q}{e} &= \frac{e^2}{C^\varepsilon} u \end{aligned} , \quad (1)$$

where u represents the mechanical displacement, q is the electric charge displacement and f is the excitation force. The constants m, K^E, L, R, C^ε and e are the mass, the stiffness in short circuit, the inductance, the resistance, the piezoelectric capacitance and the coupling coefficient, respectively. For a linear mechanical structure, the use of a linear resonant shunt tuned to its optimal inductance and resistance values [2, 3] (see Figure 1(a)) offers a vibration damping solution whose performance does not depend on the excitation amplitude. However, when a nonlinear stiffness, e.g., $K^E = K_0^E + K_{\text{NL}} u^2$, is present the linear resonant shunt looses its efficiency, as shown in Figure 1(b) [4, 6]. A NPTVA is thus required in order to mitigate the nonlinear vibrations.

Principle of similarity through nonlinear capacitance

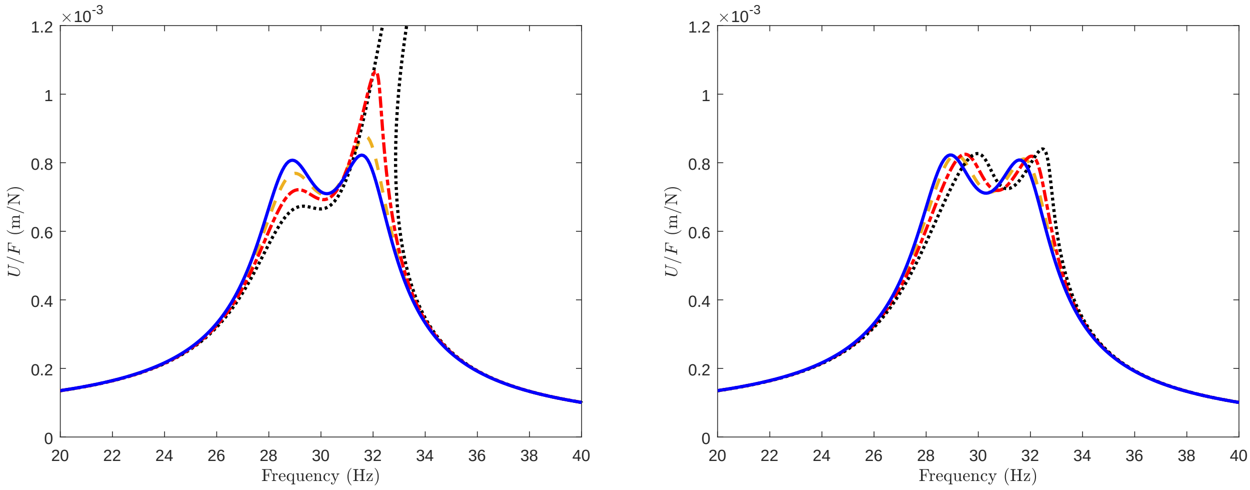
As shown by Soltani et al. [6], the introduction of a nonlinear component into the piezoelectric tuned vibration absorber reduces the effect of the nonlinearity of the primary system. Based on a principle of similarity, it was proposed to realize a nonlinear capacitance that is the electrical analog of the mechanical nonlinearity of the primary system. A cubic nonlinearity in the mechanical domain thus requires an electrical component providing a voltage proportional to q^3:

$$\begin{aligned} m\ddot{u} + \left(K_0^E + \frac{e^2}{C^\varepsilon}\right) u &= f + \frac{e^2}{C^\varepsilon}\frac{q}{e} - K_{\text{NL}} u^3 \\ e^2 L_0 \frac{\ddot{q}}{e} + e^2 R \frac{\dot{q}}{e} + \frac{e^2}{C^\varepsilon}\frac{q}{e} &= \frac{e^2}{C^\varepsilon} u - K_{\text{C}} \left(\frac{q}{e}\right)^3 \end{aligned} . \quad (2)$$



(a) (b)

Figure 1: Displacement of a structure coupled to a linear piezoelectric tuned vibration absorber, in short-circuit configuration (· · ·) or with an optimal linear shunt (—): (a) linear host structure, (b) nonlinear host structure.



(a) (b)

Figure 2: Simulated frequency response functions for various forcing amplitudes, $F = 0.2$ N (—), $F = 0.4$ N (– –), $F = 0.6$ N (– · –) and $F = 0.8$ N (· · ·): (a) linear shunt, (b) NPTVA that satisfies Equation (3).

K_C corresponds to a constant characterizing the nonlinear capacitor for which the optimal tuning was proposed in [6]. For a sufficiently low "mass" ratio e^2L_0/m, the tuning rule can be approximated by

$$K_{\mathrm{C}} \simeq 2\left(\frac{e^2L_0}{m}\right)^2 K_{\mathrm{NL}}. \tag{3}$$

When K_{C} is equal to zero, the resonant shunt is linear, and Figure 2(a) shows that an increase of the excitation amplitude leads to an inadequate tuning of the vibration absorber because of the varying mechanical resonance. High forcing amplitudes even lead to a merging of the right resonance peak with an isolated resonance curve similarly to what occurred in Figure 1(b). Considering a NPTVA that satisfies Eq. (3), Figure 2(b) depicts a mechanical behavior close to the fully linear case, i.e., equal peaks can be maintained, which demonstrates the efficiency of the NPTVA. Similar results can be obtained for other types of nonlinearity as long as the principle of similarity is considered [4, 6].
Considering a single resonance, it can be shown that a one-term harmonic balance approximation is usually enough [6]. We thus assume

$$q = Q\sin(\omega t - \delta) \quad \text{and} \quad \sin^3(\omega t - \delta) \simeq \frac{3}{4}\sin(\omega t - \delta), \tag{4}$$

where Q is the electric charge displacement amplitude, ω is the angular frequency and δ is the phase constant. From

Eqs. (2) and (4), we define the nonlinear voltage

$$v_{\mathrm{NL}} = \frac{K_{\mathrm{C}}}{e}\left(\frac{q}{e}\right)^3 = \frac{3K_{\mathrm{C}}}{4e^4}Q^3\sin(\omega t - \delta). \tag{5}$$

Considering the one-term harmonic balance approximation, the previous equation is equivalent to

$$V_{\mathrm{NL}} = \frac{3K_{\mathrm{C}}}{4e^4\omega^3}\dot{Q}^3, \tag{6}$$

where V_{NL} represents the amplitude of the nonlinear voltage, and $\dot{Q} = \omega Q$ is the amplitude of the electrical current. Equation (6) thus describes the nonlinear electrical component that needs to be incorporated into the resonant shunt in order to compensate the mechanical nonlinearity through a principle of similarity.

Principle of similarity through nonlinear inductance

Capacitors offering a nonlinear relation between voltage and current can be found, but they exhibit a non-negligible linear contribution [7]. Another solution could be to use a synthetic impedance, but it requires a power supply, which may be bulky and does not prevent from potential stability issues. The solution considered herein is to exploit an inductor whose equivalent inductance value decreases with an increase of the electrical current $\dot{q}$ as

$$L(\dot{q}) = L_0\left(1 - \alpha\dot{q}^2\right). \tag{7}$$

which generates a hardening nonlinearity. Such a dependence can be exhibited by passive inductors subjected to magnetic saturation. Considering the inductance in Eq. (7), the coupled system in Eq. (1) becomes

$$\begin{array}{c} m\ddot{u} + \left(K_0^E + \dfrac{e^2}{C^\varepsilon}\right)u = f + \dfrac{e^2}{C^\varepsilon}\dfrac{q}{e} - K_{\mathrm{NL}}u^3 \\ e^2L_0\dfrac{\ddot{q}}{e} + e^2R\dfrac{\dot{q}}{e} + \dfrac{e^2}{C^\varepsilon}\dfrac{q}{e} = \dfrac{e^2}{C^\varepsilon}u + eL_0\alpha\dot{q}^2\ddot{q} \end{array}. \tag{8}$$

Adopting a one-term harmonic balance approximation, the nonlinear term in the electrical domain is

$$\dot{q}^2\ddot{q} = -\left(Q\omega\cos(\omega t - \delta)\right)^2\omega^2 Q\sin(\omega t - \delta) = -\omega^4Q^3\left(\sin(\omega t - \delta) - \sin(\omega t - \delta)^3\right) \tag{9}$$

So, the corresponding nonlinear voltage in the resonant shunt is

$$v_{\mathrm{NL}} = -L_0\alpha\dot{q}^2\ddot{q} = \frac{L_0\alpha\omega^4}{4}Q^3\sin(\omega t - \delta), \tag{10}$$

which gives a relation between the nonlinear voltage amplitude and the electrical current amplitude

$$V_{\mathrm{NL}} = \frac{L_0\alpha\omega}{4}\dot{Q}^3. \tag{11}$$

Considering that around a resonance, the angular frequency ω can be seen as almost constant, a cubic relation similar to Eq. (6) is obtained if we ensure that

$$\frac{L_0\alpha\omega}{4} = \frac{3K_{\mathrm{C}}}{4e^4\omega^3} \quad \text{so} \quad \alpha = 6\frac{L_0K^{\mathrm{NL}}}{m^2\omega^4}. \tag{12}$$

It thus turns out that a NPTVA with saturable inductor (with quadratic decrease) can effectively mitigate the vibrations of a system with cubic mechanical nonlinearity.

Experimental validation

Piezoelectric damping with a linear shunt

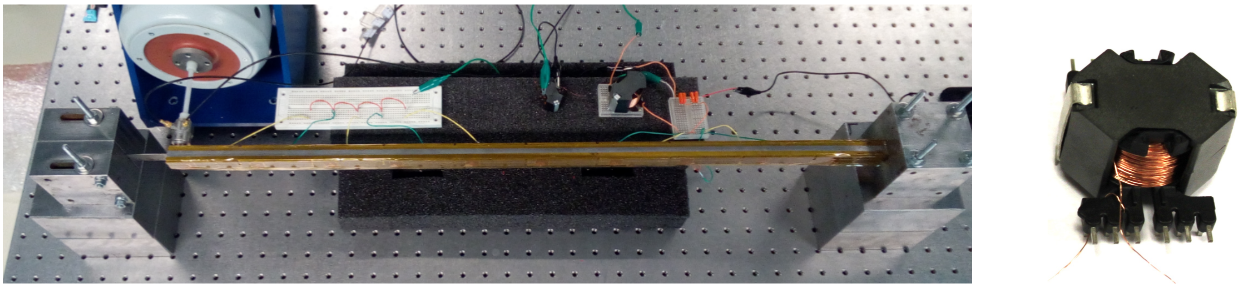
The experimental setup is made of a 700 mm long cantilever beam [5, 8, 9]. The mechanical nonlinearity is due to a thin lamina located at the free end of the beam, as shown in Figure 3(a). The structure is covered with an array of piezoelectric patches connected in parallel. The first step is to design a passive inductor that realizes a linear tuned vibration absorber when coupled to the piezoelectric patches. By measuring the short-circuit and open-circuit angular frequencies at resonance, ω_{S} and ω_{O} respectively, the coupling ratio is computed as

$$k_{\mathrm{c}} = \sqrt{\frac{\omega_{\mathrm{O}}^2 - \omega_S^2}{\omega_S^2}} = 0.124. \tag{13}$$

From measurements at low amplitudes, it is also possible to extract the modal constants m, K_0^E and the coupling coefficient e. As the total piezoelectric capacitance is around $C^\varepsilon = 240$ nF and the first open-circuit resonance frequency is 30.3 Hz, the optimal shunt inductance and resistance can be approximated by

$$L_0 = \frac{1}{C^\varepsilon\omega_{\mathrm{O}}^2} = 115\ \mathrm{H} \quad \text{and} \quad R = \sqrt{\frac{3}{2}}\frac{k_{\mathrm{c}}}{C^\varepsilon\omega_{\mathrm{O}}} = 3300\ \Omega. \tag{14}$$

A passive inductor with a magnetic circuit in ferrite material (see Figure 3(b)) can satisfy the previous requirements [10]. When the inductor is connected to the piezoelectric patches, Figure 4 shows a 23 dB vibration reduction compared to a case with no shunt.



(a) (b)

Figure 3: Experimental setup: (a) cantilever beam with additional clamping through a thin lamina (left end), (b) physical inductor made of a magnetic core in ferrite.

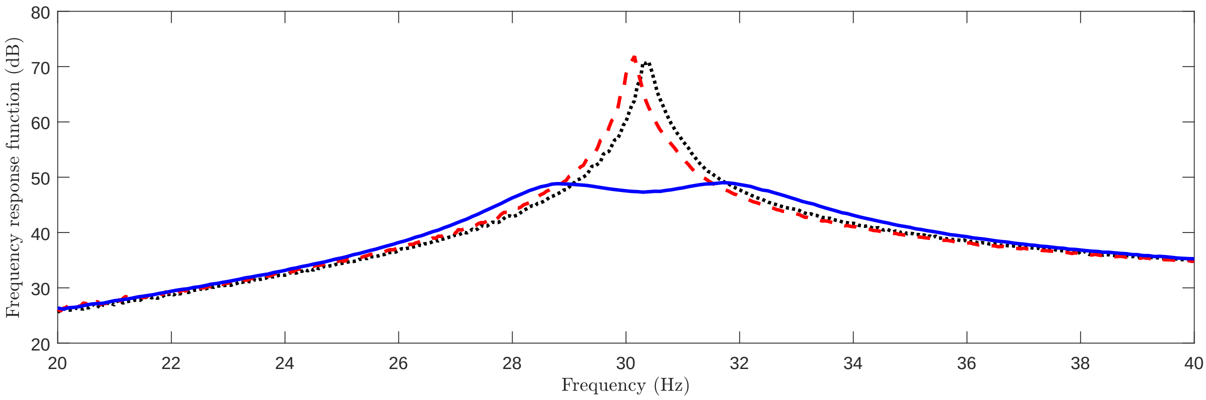


Figure 4: Frequency response function around the first mechanical resonance, with short circuit (···), with open circuit (···) or with an optimal linear shunt (—).

Design of a nonlinear inductor

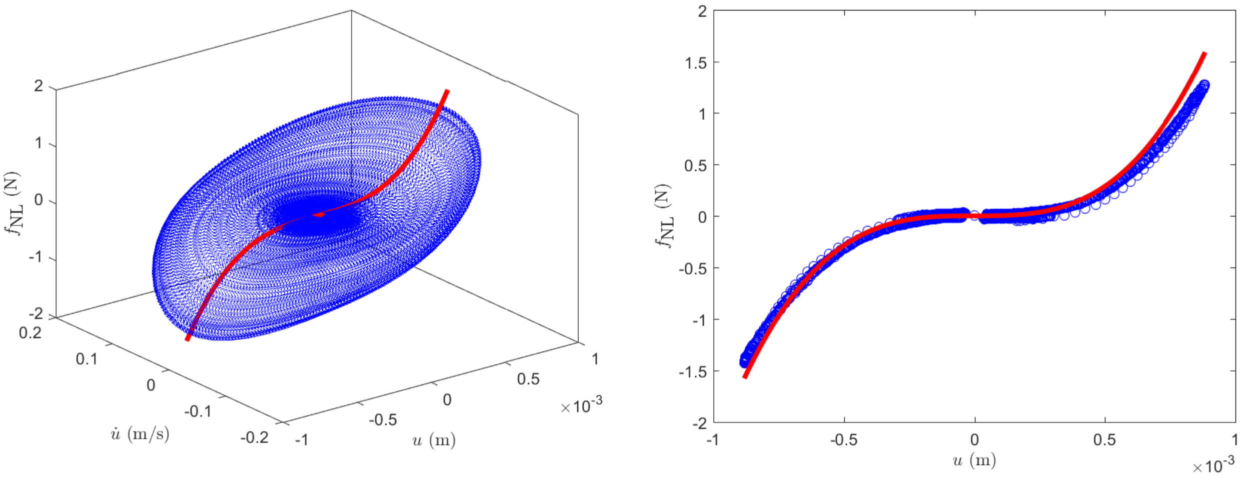
To determine the mechanical nonlinearity, the restoring force surface method is used through which the nonlinear restoring force $f_{\mathrm{NL}} = f - m\ddot{u} - K_0^E u$ is computed from temporal series. Figure 5 represents its phase space representation (3D and 2D section) that displays that the nonlinearity cannot be neglected for displacements above 0.1 mm. Figure 5(b) shows that the nonlinear restoring force can be approximated by a cubic term $K^{\mathrm{NL}}u^3$ whose constant $K^{\mathrm{NL}} = 2.3 \times 10^9$ N/m^3 is obtained from the experimental results.

Based on the knowledge of the nonlinear coefficient, an inductor with magnetic saturation that generates a current dependence sufficiently close to that given in Eq. (7) is designed. For convenience, two inductors are used for the present nonlinear shunt, i.e., a suitable saturable inductor is first selected and completed by an additional linear inductor. The actual nonlinearity of the electrical components is measured experimentally by computing a nonlinear restoring voltage $v_{\mathrm{NL}} = v - L\ddot{q} - R\dot{q}$, where v is an excitation voltage applied to the component. The result is a dependence of the voltage amplitude on the electrical current, as represented in Figure 6(a). The variation of the corresponding equivalent inductance is given in Figure 6(b). Even if the inductance and the corresponding voltage do not offer pure quadratic and cubic nonlinearities, respectively, the experimental results reveal that they are a sufficiently good approximation over the considered range of excitation.

Performance of the nonlinear shunt

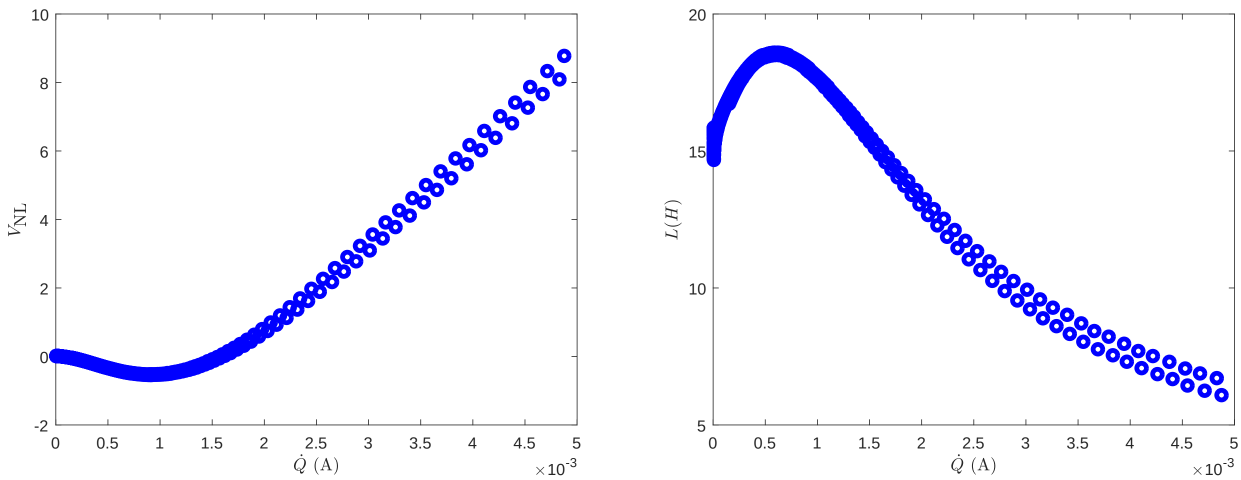
The experimental results in Figure 7(a) first show the limits of the linear shunt, which was tuned in order to obtain an equal-peak configuration at low forcing amplitude ($F = 0.2$ N). In accordance with Figure 2(a), increasing forcing amplitudes generates a detuning of the absorber, which becomes critical when F is equal or above 0.6 N. The case $F = 0.8$ N could not be tested, because of the too high amplitudes at the end of the beam, in agreement with the simulated results presented in Figures 1(b) and 2(a).

On the other hand, Figure 7(b) depicts that the NPTVA with a saturable inductor is able to maintain equal peaks in the considered range of forcing amplitudes. This is due to the decrease in the equivalent inductance value for increasing forcing amplitudes, hence the electrical resonance frequency is able to follow the increase in the mechanical resonance frequency. This is confirmed by the local minima of the frequency response functions which are moved toward higher frequencies. A final observation is that the amplitudes of the peaks in the FRFs do not present a dependence on forcing amplitude, as if the coupled system was obeying the superposition principle.



(a) (b)

Figure 5: Experimental nonlinear restoring force (o) and cubic approximation (—): (a) 3D representation as a function of u and $\dot{u}$, (b) restoring force when $\dot{u} = 0$.



(a) (b)

Figure 6: Electrical measurements on the inductor: (a) nonlinear voltage, (b) equivalent inductance value.

Conclusions

The main objective of this study is to realize a fully passive nonlinear piezoelectric vibration absorber that is tuned according to the principle of nonlinear similarity. The proposed solution involves a saturable inductor whose equivalent inductance value depends on the electrical current flowing through the shunt. This results in a variable electrical resonance frequency that compensates the mechanical nonlinearity. Compared to a linear tuned vibration absorber, the nonlinear shunt is able to maintain an adequate tuning over a wider range of forcing amplitudes. Eventually, the proposed nonlinear piezoelectric tuned vibration absorber improves vibration mitigation without any need of power supply or external controller.

References

[1] Hagood, N. W., von Flotow, A. (1991). Damping of structural vibrations with piezoelectric materials and passive electrical networks. *J. Sound Vib.*, **146**:243–268.

[2] Thomas, O., Ducarne, J., Deü, J.-F. (2012). Performance of piezoelectric shunts for vibration reduction. *Smart. Mater. Struct.* **21**:015008

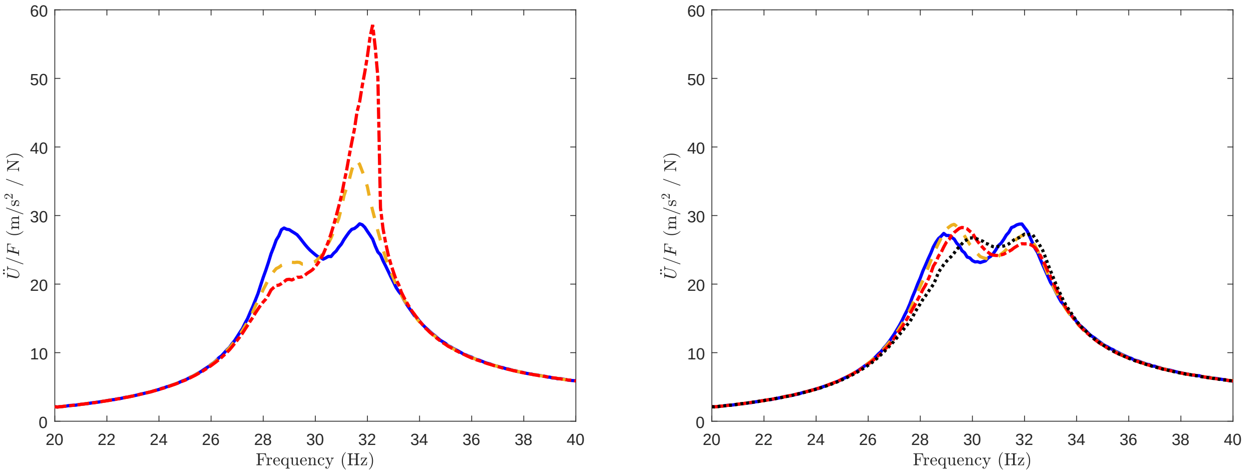
[3] Soltani, P., Kerschen, G., Tondreau, G., Deraemaeker, A. (2014). Piezoelectric vibration damping using resonant shunt circuits: an exact solution. *Smart Mater. Struct.*, **23**:125014.

[4] Habib, G., Kerschen, G. (2016). A principle of similarity for nonlinear vibration absorbers. *Physica D* **332**:1–8.

[5] Habib, G., Grappasonni, C., Kerschen, G. (2016). Passive linearization of nonlinear resonances. *J. Appl. Phys.* **120**:044901.

[6] Soltani, P., Kerschen, G. (2015). The nonlinear piezoelectric tuned vibration absorber. *Smart. Mater. Struct.* **24**:075015.

[7] Gluskin, E. (1985). The use of non-linear capacitors. *Int. J. Electron.*, **58**:63–81.



(a) (b)

Figure 7: Experimental frequency response functions for various forcing amplitudes, $F = 0.2$ N (—), $F = 0.4$ N (– –), $F = 0.6$ N (– · –) and $F = 0.8$ N ($\cdots$): (a) linear resonant shunt, (b) NPTVA with a saturable inductor.

[8] Thouverez, F. (2003). Presentation of the ECL benchmark. *Mech. Syst. Signal Process.*, **17**:195–202.

[9] Grappasonni, C., Habib, G., Detroux, T., Kerschen, G. (2016). Experimental demonstration of a 3d-printed nonlinear tuned vibration absorber. *In Nonlinear Dynamics*, **1**:173–183.

[10] Lossouarn, B., Aucejo, M., Deü, J. F., Multon, B. (2017). Design of inductors with high inductance values for resonant piezoelectric damping. *Sens. Actuators A*, **259**:68–76.