Flutter instability of a visco-elastic belt drive

Alois Steindl*
Institute for Mechanics and Mechatronics, TU Wien, Vienna, Austria

Summary. We investigate the behaviour of a visco-elastic planar belt drive, which is driven by a steadily rotating drum. Due to the presence of the small damping parameter and the small bending stiffness of the belt, the equations of motion are singularly perturbed: Close to the separation from the drums the curvature varies significantly and the precise separation point has to be determined by the boundary layer equations.
In the first step we derive the ordinary differential equations for the steady configuration of the constantly driven belt. Along this steady configuration we linearize the equations of motion and calculate the stability of the configuration by determining the critical eigenvalues and corresponding modes. By varying the drive speed, the damping coefficient, the tension force and the radii of the drums, we locate the stability boundary for the steady motion in parameter space. Preliminary calculations indicate, that the damping parameter has a strong influence on the critical drive speed.

Introduction and model setup

Drive belts are frequently used tools for power transmission and their stable behaviour is important for the proper operation of the facility. For slowly moving belts it is usually sufficient to consider the equilibrium states, but if the belt speed approaches the wave speed in the belt, the influence of the drive motion has to be taken into account. In this talk we investigate the onset of flutter, when the drive speed is increased quasistatically and its dependence on the viscous material damping, the tension force in the belt and the curvatures of the driving and driven drum.

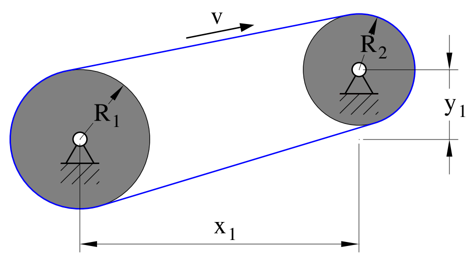

Figure 1: Mechanical model of the belt drive moving with velocity v

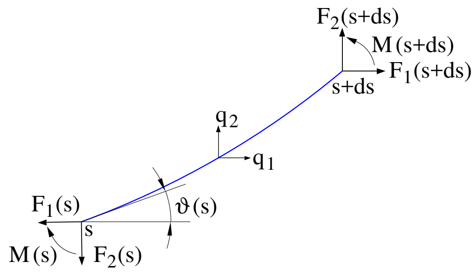

Figure 2: Cross-sectional forces F_i, bending moment M, and distributed forces q_i acting on a beam segment

A simple model for the drive is displayed in Fig. 1: We assume, that the driving drum has radius R_2 and rotates in clockwise direction. Furthermore, we assume, that there is no slip between the belt and the drum. Since in this configuration the tension in the upper span of the belt is larger, we restrict our attention to this segment.
Before deriving the equations for the belt, we rescale the lengths by a reference length L_0, like the distance between the centers of the drum, the (generalized) forces by the pretension P_0, and the time by $\sqrt{\varrho A L_0^2/P_0}$, such that the wave speed in the belt is scaled to unity. Assuming a Kelvin-Voigt material law and an extensible Euler-Bernoulli beam model for the belt, we obtain the differential equations for a line segment displayed in Fig. 2, (see e.g. [1])

$$x' = (1+\varepsilon)\cos\vartheta, \qquad y' = (1+\varepsilon)\sin\vartheta, \tag{1a}$$
$$\varepsilon + \delta\dot{\varepsilon} = \beta N = \beta(F_1\cos\vartheta + F_2\sin\vartheta), \tag{1b}$$
$$\gamma(\vartheta' + \delta\dot{\vartheta}') = M, \tag{1c}$$
$$M' = (1+\varepsilon)(F_1\sin\vartheta - F_2\cos\vartheta), \tag{1d}$$
$$F_1' = -q_1 + \ddot{x}, \qquad F_2' = -q_2 + \ddot{y}. \tag{1e}$$

Here $(\cdot)'$ denotes the derivative w.r.t. the rescaled unstrained arc length s, ϑ is the inclination angle, δ is the damping parameter, $\beta = P_0/EA$ is the elongation due to pre-stretching and $\gamma = EJ/(P_0 L_0^2)$ is the bending stiffness. For typical belts the parameters δ, β and γ are very small.
In (1) the arc-length s denotes the length from a certain material point of the belt. In order to study steady motions, we set $s = S + vt$, where the new variable S denotes the length along a steady configuration. With

$$\frac{\partial u(s,t)}{\partial s} = \frac{\partial u(S+vt,t)}{\partial S} \quad \text{and} \quad \frac{\partial u(s,t)}{\partial t} = \frac{\partial u(S+vt,t)}{\partial t} + \frac{\partial u(S+vt,t)}{\partial S}v$$

the time derivatives $\dot{u}$ in (1) have to be replaced by $vu' + \dot{u}$, where the prime now denotes the derivative w.r.t. S. Therefore the material laws and the equilibrium equations in (1) become

$$\varepsilon + \delta(v\varepsilon' + \dot{\varepsilon}) = N, \tag{1b'}$$

$$\gamma(\vartheta' + \delta(v\vartheta'' + \dot{\vartheta}')) = M, \tag{1c'}$$

$$F_1' = -q_1 + v^2 x'' + 2v\dot{x}' + \ddot{x}, \qquad F_2' = -q_2 + v^2 y'' + 2v\dot{y}' + \ddot{y}. \tag{1e'}$$

The boundary conditions at the endpoints state, that the free portion of the belt starts and ends at the circumference of the drums and that the curvature is given by the radius of the drums. Further the time derivatives of ε and M vanish at the left drum and the belt length remains constant

$$x(0) = -R_1 \sin\vartheta(0), \qquad x(L) = x_1 - R_2 \sin\vartheta(L), \tag{2a}$$

$$y(0) = R_1 \cos\vartheta(0), \qquad y(L) = y_1 + R_2 \cos\vartheta(L), \tag{2b}$$

$$\vartheta'(0) = -1/R_1, \qquad \vartheta'(L) = -1/R_2, \tag{2c}$$

$$\varepsilon(0) = \beta N(0), \qquad \gamma\vartheta'(0) = M(0), \tag{2d}$$

$$-R_1\vartheta(0) + L + R_2\vartheta(L) = L_{\text{fix}}, \tag{2e}$$

where L_{fix} denotes the unstretched cable length between the tops of both drums. It is initially calculated by requiring $N(L) = 1$ for the undriven belt $v = 0$.

Preliminary results

By assuming a straight initial configuration, neglecting the weight, geometric nonlinearities and the elongation, and stating simply supported boundary conditions, we obtain the linear differential equation ([1])

$$\ddot{y} + 2v\dot{y}' + (v^2 - 1)y'' + \gamma(y'''' + \delta(vy' + \dot{y})'''') = 0, \tag{3a}$$

$$y(0) = 0, \qquad y(1) = 0, \tag{3b}$$

$$y''(0) = 0, \qquad y''(1) = 0, \tag{3c}$$

$$vy'''(0) + \dot{y}''(0) = 0. \tag{3d}$$

The evolution of the lowest order eigenvalues over the belt velocity is displayed in Fig. 3: The undamped system first

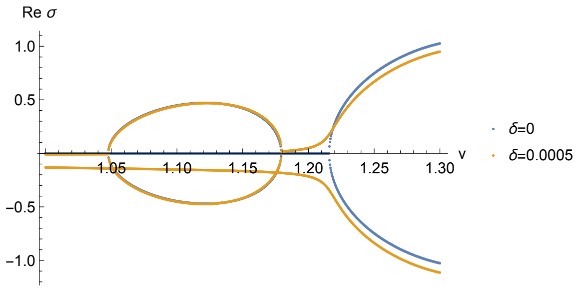

Figure 3: Evolution of the real part of the lowest order eigenvalues of (3) over v for the damped and undamped system

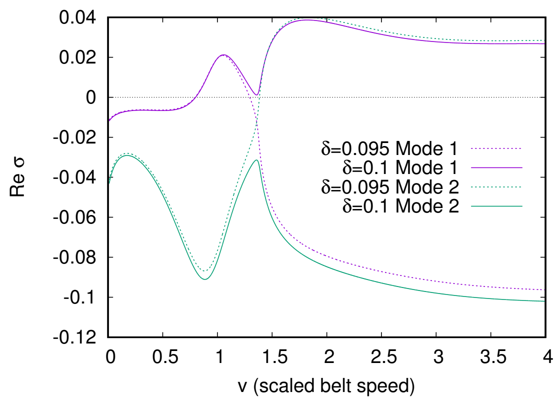

Figure 4: Mode switching at the Hopf bifurcation

becomes unstable by a pair of real eigenvalues, but then becomes stable again. The pair of purely imaginary eigenvalues encounters a second imaginary pair and undergoes a Hamiltonian Hopf bifurcation ([2, 3]). In the presence of viscous damping the trivial state is first asymptotically stable, then undergoes a divergence bifurcation and remains unstable.
In Fig. 4 the variation of the real part of the leading two eigenvalues is displayed for the full problem (1) with moderately small drums ($R = 0.2$) and a certain set of parameters and varying belt speed v: For $\delta = 0.1$ the first mode becomes unstable at $v \approx 0.8$ and remains unstable, whereas the second mode is stable throughout. If the damping parameter δ is descreased by 5%, the first mode becomes stable again close to $v = 1.4$, but now the second mode becomes unstable.

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