Mass detection through parametric analysis and symmetry-breaking in a MEMS array

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Summary. Due to their low cost, size and precision M/NEMS are efficient sensors. M/NEMS sensors are used in various domains ranging from aeronautics to medicine or telecommunication, with applications such as chemical, inertial or mass sensing. Our previous researches on mass sensing were focused on a single resonator. In this work, a symmetric array of three resonant nanobeams is analysed. The originality lies in the use of a direct parametric analysis to sense an added mass during a symmetry-breaking event.

Array of three nanomechanical resonators

Let a 3-beams-array be considered and sketched in Fig. 1. Each beam constitutes an electrostatic actuator for its adjacent beams. All the beams share the same geometrical parameters and material properties: length l, width b, height h, Young's modulus E, moment of inertia I, material density ρ, gap g between two adjacent beams.

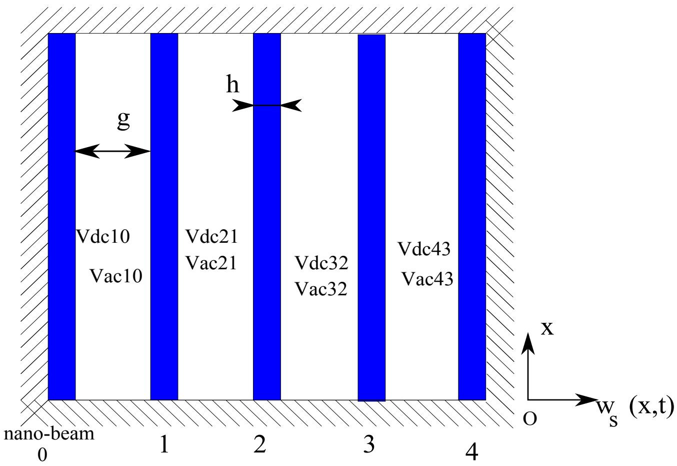


Figure 1: Array of three clamped-clamped M/NEMS beams.

The s^{th} beam is actuated by electrostatic forces generated by the adjacent beams. $V_{s,s+1} = V_{dcs,s+1} + V_{acs,s+1}\cos(\Omega t)$ constitutes the voltage applied between the successive beams s and $s+1$, with V_{dc}, V_{ac} the continuous and alternative voltage respectively. The resulting equations of motion take the following form [1] [2]

$$EI\frac{\partial^4 \tilde{w}_s(\tilde{x},\tilde{t})}{\partial \tilde{x}^4} + \rho bh\frac{\partial^2 \tilde{w}_s(\tilde{x},\tilde{t})}{\partial \tilde{t}^2} - \Big[\tilde{N} + \frac{Ebh}{2l}\int_0^l \big(\frac{\partial \tilde{w}_s(\tilde{x},\tilde{t})}{\partial \tilde{x}}\big)^2 \mathrm{d}\tilde{x}\Big]\frac{\partial^2 \tilde{w}_s(\tilde{x},\tilde{t})}{\partial \tilde{x}^2} = \frac{\epsilon_0 bC_n}{2}\left[\frac{V_{s,s+1}^2}{\left(g+\tilde{w}_{s+1}-\tilde{w}_s\right)^2} - \frac{V_{s-1,s}^2}{\left(g+\tilde{w}_s-\tilde{w}_{s-1}\right)^2}\right] \quad (1)$$

where $s = 1,..,3$ and ϵ_0, C_n are the dielectric constant and fringing field coefficient respectively. $\tilde{N}_s$ represents the axial preload along the x-axis set by design or due to manufacturing process. Beams $0, 4$ on both extremities of the array are totally clamped, the other beams $1,..,3$ are clamped-clamped. Therefore the boundary conditions are given by (2).

$$\begin{gathered}\tilde{w}_0(\tilde{x},\tilde{t}) = \tilde{w}_{n+1}(\tilde{x},\tilde{t}) = 0\\ \tilde{w}_s(0,t) = \tilde{w}_s(1,t) = \frac{\partial \tilde{w}_s}{\partial x}(0,t) = \frac{\partial \tilde{w}_s}{\partial x}(1,t) = 0\\ \tilde{w}_0(x,t) = \tilde{w}_{n+1}(x,t) = 0\end{gathered} \quad (2)$$

Since the resonators of the 3-beams-array are identical, the beams $1,..,3$ have the same undamped linear eigenmodes. Therefore these modes are used as a basis for the Galerkin method in order to remove the spatial dependence from the equations of motion (1) and to obtain a reduced order model [3]. The resulting nonlinear differential system of equations is then solved by means of the Harmonic Balance Method. The obtained periodic solutions are followed with the pseudo-arc length method in order to obtain the response curves. Characterisation of bifurcations presented in [4] are then used to detect bifurcation points such as Limit Points (LP) on the response curves. These bifurcations points can then be directly followed to carry out a parametric analysis.

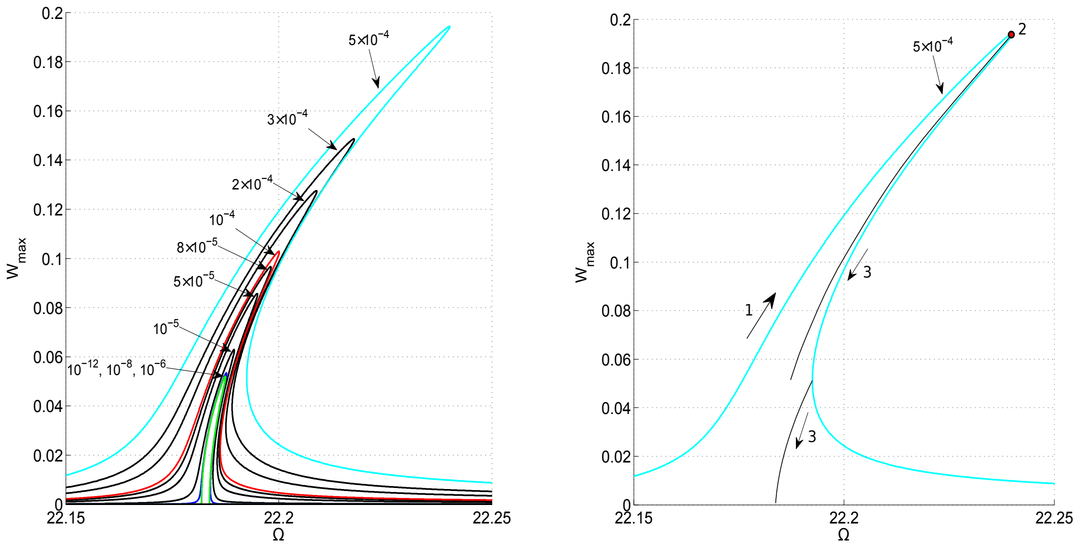
Mass Sensing by symmetry-breaking and Limit Points continuation

In order to use the symmetry-breaking event to detect and measure an added mass, the voltages are chosen as in Tab. 1. Since the voltages are symmetric, the electrostatic forces acting on the central beam compensate each other. Thus the central beam stays at rest until an added mass breaks the symmetry. When the symmetry-breaking event appears, the central beam will start vibrating. For sufficiently high added mass or applied voltages, LP bifurcations will be present within the dynamical response [4]. The induced hysteresis cycle is then used to detect the added mass [2].

Vdc_{10}	Vac_{10}	Vdc_{21}	Vac_{21}	Vdc_{32}	Vac_{32}	Vdc_{43}	Vac_{43}
0	0	5.3	1	5.3	1	0	0

Table 1: Voltage configuration of the three beams array.

In order to measure the added mass, a parametric analysis is performed. In Fig. 2a several responses curves for different non-dimensional added masses have been computed, the non-dimensional mass m being defined as $m_p/\rho bh$ with m_p the physical value of the added mass and ρbh the mass of one beam. However, this method is time consuming and is not accurate. Instead of computing responses curves for several different added masses, it is more judicious to detect a starting LP for a given value of the added mass (point 2 in Fig 2b) and then directly follow the branch of LP [4] with respect to the added mass. Doing so, the parametric analysis is performed with only one calculation and the curve of LP in Fig. 2b provides the amplitude of the central beam after the symmetry-breaking induced by an added mass.



(a) Responses curves for multiple added masses

(b) Limit-point tracking procedure. 1: Response Curve, 2: Limit Point Localisation, 3: Limit-point tracking

Figure 2: Response curves of the central resonator for a varying non-dimensional added mass.

Conclusions

A parametric analysis of a three-resonator array has been performed, based on a symmetry-breaking event in order to detect and measure an added mass. The parametric analysis based on the direct continuation of Limits Points constitutes a very efficient method for the numerical estimation of the measured mass with only one calculation. This research represents a step towards the implementation of MEMS-based mass spectrometry using arrays of resonators.

References

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