Impulsive damping of mechanical systems: periodic solutions and energy harvesting

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Summary. In the present contribution, periodic impulsive damping is introduced with the aim to extract energy from externally excited mechanical systems. It is shown that periodic solutions exist, which allow to extract much more energy from the mechanical system than an optimized passive one. The influence of the impulsive strength and the frequency of the external excitation to the extracted energy are investigated.

Introduction

Affecting the energy content of mechanical systems in a targeted manner is the focus of all measures for vibration control or energy harvesting, and hence, a variety of active and passive methods have been developed. Most of them are based on continuously influencing the mechanical system. For example, energy harvesting by introducing time-periodic damping of a single-degree of freedom system is investigated in [1]. By contrast, the effect of stiffness impulses to the modal energy content of mechanical systems was investigated in [2]. It was demonstrated that therewith, a transfer of vibration energy across modes is possible, which finally results in a much faster decay of vibration amplitudes.
In the present contribution, periodic impulsive damping is introduced with the aim to extract energy from externally excited mechanical systems. It is shown that periodic solutions exist, which allow to extract much more energy from the mechanical system than an optimized passive system.

Analytical Investigations

Figure (1) shows a sketch of the investigated mechanical system, consisting of a base excited non-linear single degree of freedom oscillator with mass m, constant stiffness k and time-varying damping coefficient $c(t)$.

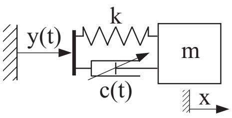

Figure 1: Sketch of investigated mechanical system.

The time-varying damping $c(t)$ is assumed to be of the form

$$c(t) = c_I + c_P \left\{ \sum_{k=1}^{K} \varepsilon_k \delta(t - t_k) \right\}, \tag{1}$$

where c_I = const., and $\delta(t - t_k)$ represents the Dirac delta function. Furthermore, it is assumed that the impulsive damping is time-periodic, i.e. the time span $\Delta t = t_{k+1} - t_k$, $k = 1 \dots K$, between adjacent impulses is constant, as well as the impulsive strength $c_P \varepsilon_k = c_P \varepsilon$. Introducing the relative displacement $z = x - y$ allows to write the equations of motion in the form

$$m\ddot{z} + c(t)\dot{z} + kz = -m\ddot{y}, \tag{2}$$

where $y(t) = Y \sin(\Omega t)$ represents the external base excitation. The relation between the state-vector $\mathbf{z}_{k-1,+}$ just after an impulse applied at the instant of time t_{k-1}, and the state-vector $\mathbf{z}_{k,-}$ just before the following impulse is obviously given by

$$\mathbf{z}_{k-} = \mathbf{D}(\Delta t)\mathbf{z}_{k-1,+} + \mathbf{z}_{e,k}, \tag{3}$$

where the first term represents the zero-input response, and the second one the zero-state response obtained from solving the convolution integral. The state-vector just after a parametric impulse is given by $\mathbf{z}_{k+} = \mathbf{J}\mathbf{z}_{k-}$, see Hsu [3], where $\mathbf{J} = \mathbf{e}^{\hat{\mathbf{A}}}$ is denoted as jump-transfer matrix. Therein, the matrix $\hat{\mathbf{A}}$ contains the parameters with impulsive behaviour. In the present case where damping impulses are applied $\hat{\mathbf{A}} = \mathrm{diag}(0, -\varepsilon c_P/m)$ holds, i.e. only the velocity $\dot{z}$ is subject of variation, whereas the displacement z remains unchanged across an impulse, see also [4]. If a sequence of p impulses is applied, the state vector after the last impulse is given by

$$\mathbf{z}_{p+} = (\mathbf{JD})^p \mathbf{z}_{0+} + \underbrace{\sum_{r=0}^{p-1} (\mathbf{JD})^r \mathbf{J}\mathbf{z}_{e,p-r}}_{\mathbf{z}_{e,p}} . \tag{4}$$

In the following, it is assumed that an equidistant number of p impulses is applied within N periods of external excitation. A corresponding steady-state solution $\mathbf{z}_p^{(N)}$ satisfies

$$\mathbf{z}_p^{(N)} = (\mathbf{JD})^p \mathbf{z}_p^{(N)} + \mathbf{z}_{e,p}, \tag{5}$$

which is unique if $\mathbf{I} - (\mathbf{JD})^p$ is non-singular, where $\mathbf{I}$ represents the identity matrix. In this case, the steady-state solution is given by

$$\mathbf{z}_p^{(N)} = (\mathbf{I} - (\mathbf{JD})^p)^{-1} \mathbf{z}_{e,p}, \tag{6}$$

which is globally attracting if the absolute values of all eigenvalues of $(\mathbf{JD})^p$ are less than one. The variation of the total energy of the mechanical system after N periods of external excitation,caused by the parametric impulses, is given by $E_{ex,p}^{(N)} = \frac{1}{2}m\sum_k^{k+p-1}(\dot{z}_{k+}^2 - \dot{z}_{k-}^2)$. For the sake of comparison, a passive system is introduced,where the impulsive damping is replaced by a constant damping c_V. It was shown by Di Monaco et al., see [1], that an optimum value $c_{V,opt.} = 1/\Omega\sqrt{(k - m\Omega^2)^2 + (c_I\Omega)^2}$ exists, which allows to extract a maximum of energy from the mechanical system.

Numerical Results

For the following numerical results, the system parameters $m = 1$kg, $k = 1$N/m, $c_I = 0.1$Ns/m, $c_P = 1$Ns/m and $Y = 1$m are used. Figure (2) (a) shows the extracted energy in the steady-state per $N = 1$ period of external excitation, $p = 2, \ldots 5$ damping impulses and $\Omega = 1.2$, for different values of ε. One observes that for each p, an optimum value of ε exists, which allows to extract as much energy as possible, e.g. for $p = 2$, $\varepsilon_{opt} = 3.20$. The steady-state solution for $N = 1, p = 2, \varepsilon_{opt} = 3.20$ is depicted in Fig. (2) (b), demonstrating the periodic occurrence of jumps in the velocity $\dot{z}$. In Fig. (2) (c) the effect of a variation of the relative excitation frequency Ω/ω_n, where ω_n represents the natural frequency of the system, is investigated. Therefore, for each depicted case the optimum value of ε is calculated based on $\Omega = 1.2$. Additionally, a passive system is introduced, where $c_{V,opt.}$ is based on the same frequency. For $\Omega > 1.1$, the system with $N = 1, p = 2$ allows to extract much more energy than the passive system. In the vicinity of $\Omega/\omega_n = 1$, $E_{ex,2}^{(1)}$ drops down to zero. In this range, $p = 3, 4, 5$ give almost the same result than the passive system. Figure (2) (d) provides a comparison of the impulsively excited, and the passive system. The relation $\kappa = E_{ex,p}^{(N)}/E_{ex,passive}$ is depicted for $p = 2, 3$. It clearly shows that for $\Omega > 1.1$, the parametric impulses allow to extract much more energy from the mechanical system than the optimized passive damper. For example, at $\Omega/\omega_n = 2$, the extracted energy is $\kappa = 5.4$ times higher.

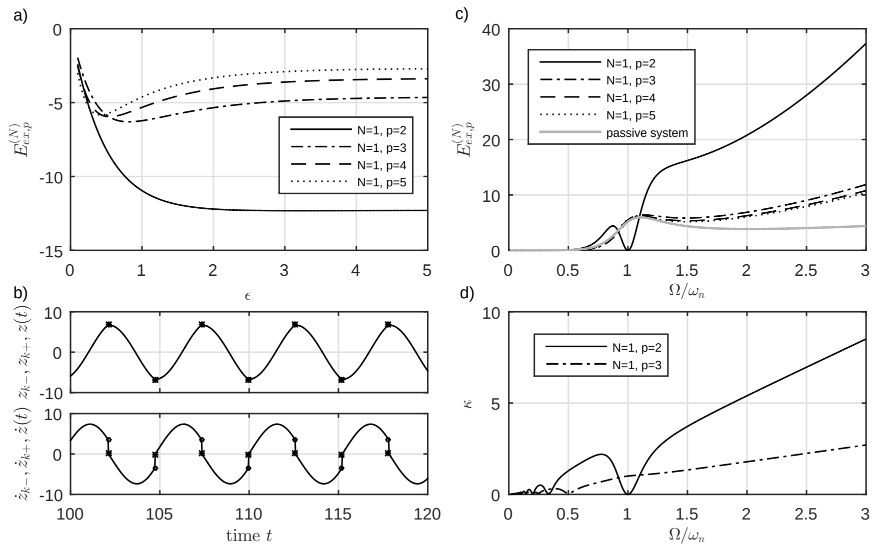

Figure 2: Extracted energy $E_{ex,p}^{N}$ by impulsive damping for different values of ε (a), timeseries of the steady-state solution in the case $N = 1, p = 2$ and $\varepsilon_{opt} = 3.20$ (b), $E_{ex,p}^{(N)}$ for variation of the frequency Ω of the external excitation and comparison with optimized passive system (c), and relative extracted energy κ for $N = 1$ and $p = 2, 3$ (d).

Conclusions

It was shown that impulsive parametric damping allows to extract energy from externally excited mechanical systems much more efficiently than an optimized passive system. Moreover, the existence of periodic solutions was demonstrated.

Acknowledgement

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