On Time-Varying Velocity for an Axially Moving String under Viscous Damping

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Summary. In this paper, an axially translating string with viscous damping and the time-varying axial velocity is studied. The time-varying velocity is assumed to be sinusoidal and small in comparison to wave velocity. The viscous damping coefficient is also assumed to be small. To solve initial-boundary value problem, a two timescales method is applied in order to obtain information whether Galerkin's truncation method is applicable for such type of problems or whether it has limited applicability in such cases. Based upon the infinite dimensional system, it has been shown that the Galerkin's method produces the wrong results for all parameter values. In order to obtain accurate approximations for long times, it has been shown that the Galerkin's method may be utilized for some particular parameter value.

Introduction

Engineering devices like conveyor belts [1], elevator cables [2], plastic films, oil pipelines [3], magnetic tapes, band-saw blades and, crane and mine hoists are considered in the category of axially translating continua. It is our common observation that there are so many advantages of these axially moving devices. The noise and vibration, particularly transversal vibration, in association to these devices have limited their applications. The understanding of transversal vibrations of axially translating continua has a lot importance in design and manufacture of these devices. Axially moving continua is categorized into string-like [4] or of beam-like [5] partial differential equations. The equations of motion for axially moving conveyor belt can be derived by Hamilton's principle. The axial velocity of a moving continua plays a significant role on the dynamics of these systems. In literature, most of the research studies are limited to a constant belt velocity. In this paper, a string-like equation is considered under viscous damping and time-varying sinusoidal axial velocity. Based upon a two timescales method, the analytical approximations of exact solutions for an boundary-initial-value problem are constructed so that the information about the complicated behavior of the system be sought.

Mathematical Model

Schematic representation of an axially translating string under viscous damping is represented in figure 1. The string is assumed to be fixed at spatial locations $x = 0$ and $x = L$. The length L is constant between the pair of pulleys. The mathematical model of an axially moving belt under viscous damping is formulated by energy principle such as Hamilton's principle. Thus, the following equation of motion with fixed boundary conditions as well as the general initial conditions is given by,

$$\rho\left(u_{tt} + 2Vu_{xt} + V_t u_x + V^2 u_{xx}\right) - Tu_{xx} + \delta\left(u_t + Vu_x\right) = 0, \quad t \geq 0, 0 < x < L \tag{1}$$

$$u(x,t) = 0, \text{at}\, x = 0, L \tag{2}$$

$$u(x,t) = f(x),\ u_t(x,t) = g(x), \text{at}\, t = 0 \tag{3}$$

where $u(x,t)$ models the transversal displacement field variable of the string, x is a spatial coordinate, t is a time, $V(t)$ is the time-varying axial velocity, ρ is the linear mass density, T is the constant tension in the string, L is the constant length between pair of pulleys and δ is the viscous damping coefficient.

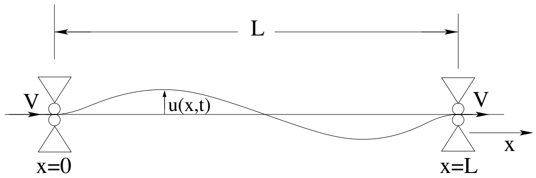

Figure 1: An axially translating belt

In order to utilize a two timescales method, we explicitly assume that the dimensional quantity δL is small in comparison to ρc, where $c = \sqrt{\frac{T}{\rho}}$. Thus, it is reasonable to express $\frac{\delta L}{\rho c} = \varepsilon\delta_0$. We also assume that the horizontal axial speed $V(t)$ is small in comparison to wave-velocity c. Thus, we can express $\frac{V(t)}{c} = \varepsilon(V_0 + \alpha\sin(\Omega t))$, where $0 < \varepsilon << 1$, V_0, α and Ω are constants. The belt always moves in forward direction only, so it is necessary to impose a condition such as $V_0 > |\alpha|$. $f(x)$ is the displacement and $g(x)$ is the velocity, both are prescribed at the initial time $t = 0$.

Method of Two Timescales

By using a two timescales method, the function $u(x,t;\varepsilon) = \bar{u}(x,t,\tau;\varepsilon)$, where $t = t$ and $\tau = \varepsilon t$. Now, by formal expansion of $\bar{u}(x,t,\tau;\varepsilon)$ in powers of ε, it yields

$$\bar{u}(x,t,\tau;\varepsilon) = \bar{u}_0(x,t,\tau) + \varepsilon\bar{u}_1(x,t,\tau) + \varepsilon^2 \cdots \tag{4}$$

By making Eqs. (1)-(3) dimensionless, then by substituting aforementioned assumptions and Eq. (4) in so-obtained equation, it follows that the $\mathcal{O}(1)$-equation is

$$\bar{u}_{0_{tt}} - \bar{u}_{0_{xx}} = 0 \tag{5}$$

and that the $\mathcal{O}(\varepsilon)$-equation is

$$\bar{u}_{1_{tt}} - \bar{u}_{1_{xx}} = -2\bar{u}_{0_{t\tau}} - 2(V_0 + \alpha\sin(\Omega t))\bar{u}_{0_{xt}} - \alpha\Omega\cos(\Omega t)\bar{u}_{0_x} - \delta_0\bar{u}_{0_t} \tag{6}$$

The solution of the $\mathcal{O}(1)$-equation is

$$\bar{u}_0(x,t,\tau) = \sum_{n=1}^{\infty}(A_{n0}(\tau)\cos(n\pi t) + B_{n0}(\tau)\sin(n\pi t))\sin(n\pi x) \tag{7}$$

where A_{n0} and B_{n0} are functions of τ and called Fourier coefficients. These unknown functions can be calculated from the $\mathcal{O}(\varepsilon)$-equation. We are interested in the bounded solutions, therefore in solution of the $\mathcal{O}(\varepsilon)$-equation it is necessary to avoid the secular terms which give rise to the unbounded terms. Thus, we collect the coefficients of $\sin(n\pi t)$ and $\cos(n\pi t)$ equal to zero, which produce system of coupled ODEs

$$\begin{aligned} \frac{dA_{n0}(\tau)}{d\tau} &= -\frac{\delta}{2}A_{n0}(\tau) + \alpha[(n-1)B_{(n-1)0}(\tau) + (n+1)B_{(n+1)0}(\tau)] \\ \frac{dB_{n0}(\tau)}{d\tau} &= -\frac{\delta}{2}B_{n0}(\tau) - \alpha[(n-1)A_{(n-1)0}(\tau) + (n+1)A_{(n+1)0}(\tau)] \end{aligned} \tag{8}$$

Now, if $nA_{n0} = X_{n0}$ and $nB_{n0} = Y_{n0}$, then the above system of coupled ODEs reduces to

$$\frac{d^2w}{d\tau^2} - 2\delta_0\frac{dw}{d\tau} - (4\alpha^2 + \delta_0^2)w = 0 \tag{9}$$

where $w(\tau) = \sum_{n=1}^{\infty}(X_{n0}^2 + Y_{n0}^2)$. Therefore, the solution to Eq. (9) is given as

$$w(\tau) = c_1\mathrm{e}^{(-\delta_0+2\alpha)\tau} + c_2\mathrm{e}^{(-\delta 0-2\alpha)\tau} \tag{10}$$

Conclusions

A two timescales method has been utilized in order to construct the formal approximations of exact solutions for the boundary-initial value problem. There are infinitely many fixed values for the harmonic frequency Ω for which internal resonances occur. Only the case for the fundamental frequency $\omega = \pi$ near to excitation frequency Ω is studied in detail. It has been shown that damping coefficient δ causes the reduction in all amplitudes of the oscillation. Furthermore, it is observed that for the case $\frac{\delta_0}{2} = |\alpha|$ the energy of the system is constant and for the case $\frac{\delta_0}{2} > |\alpha|$ the energy of the system is dissipating. Thus, in both of these cases the Galerkin's truncation method may be applicable. On the other hand, for the case $\frac{\delta_0}{2} < |\alpha|$ the energy grows exponentially and that the Galerkin's truncation method may produce wrong results.

References

[1] Sandilo, S.H., van Horssen, W.T. (2012) On Boundary Damping for an Axially Moving Tensioned Beam, *J. Vib. Acoust.* **110058**:110058-110051.

[2] Sandilo, S.H., van Horssen, W.T. (2014) On Variable Length Induced Vibrations of a Vertical String, *J. Sound. Vib.* **2449**:2449-2432.

[3] Kuiper, G.L., Metrikine, A.V. (2004) On Stability of a Clamped-Pinned Pipe Conveying Fluid, *Heron* **232**:232-2011.

[4] Malookani, R.A., van Horssen, W.T. (2015) On Resonances and the Applicability of Galerkin's Truncation Method for an Axially Moving String with Time-Varying Velocity, *J. Sound Vib.* **17**:17-1.

[5] Öz, H.R., Pakdemirli, M. (1999) Vibrations of an Axially Moving Beam with Time-Dependent Velocity, *J. Sound Vib.* **257**:257-239.