Nonlinear traffic modeling for urban road network and related robust state estimation

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Summary. The knowledge of road traffic parameters is of crucial importance to ensure state-of-the-art traffic services. Due to the widespread of today's information communication technologies the classical approach of road traffic detection has fundamentally changed. Beside or instead of traditional static traffic sensors the so called floating car data (FCD) has come to the fore. Therefore, the question arises: how to estimate traffic state from FCD information? Intermittent observations of link travel times are typical in urban road networks as the frequency of FCD based observations is time variant and traditional detector stations are not present on all links. Though, one aims to retrieve full information of the network, e.g. average travel times or traffic densities on all links. To achieve this goal a robust state estimation methodology is proposed via Kalman/H_∞ Filter which both involve data fusion and take the uncertainty of turning rates into consideration. The goal is to reconstruct the whole picture of urban road traffic from currently available measurement data. Practically, incomplete mosaic pieces of road traffic data are fused and applied to create a reliable traffic estimation both on link-level and network-level.

Introduction

As fleet management systems, autonomous vehicles and V2X technologies become even more widespread, even more FCD will be generated by them. Taxis, public transport buses and other vehicles operated by fleet management systems are sources of FCD even today. Accordingly, emerging new sets of these kinds of information (popularly called big data as well) can be efficiently exploited for traffic estimation [1]. Since data collected from different sources are heterogeneous and intermittent, a Kalman Filter based data fusion technique is proposed in this paper dealing with incomplete FCD input data, which is expected to be more appropriate than the methodologies used in current traffic state estimation systems. The available web map services typically consider only travel time data and accordingly offer information concerning the average traffic speed, e.g. Google Maps uses color codes of three states (free flow, medium, and congested traffic). Our approach, however, targets a more complex traffic estimation based on a macroscopic traffic model beside FCD travel time measurements, providing traffic density as well as average traffic speed information of all links in the network.

Applied macroscopic traffic models

Urban road network traffic modeling using link-based macroscopic fundamental diagram

Considering a macroscopic approach (individual vehicle dynamics are omitted), for link z the number of vehicles can be modeled based on the vehicle-conservation law during $[kT, (k+1)T]$ where T denotes the sample time and $k = 0,\ 1,\ 2,\ ...$ is the discrete time index:

$$n_z(k+1) = n_z(k) + T\left[\sum_{w\in I_M} \alpha_{w,z} q_w(k) - q_z(k)\right]. \quad (1)$$

The parameters in Eq. (1) are defined as follows: n_z is the number of vehicles on link z (in passenger car equivalent - PCE); I_M denotes the set of incoming links w at junction M, i.e. $w \in I_M$; $\alpha_{w,z} \in [0,1]$ is the turning rate from link w to link z; q_w denotes the traffic flow from link w (PCE/T); q_z is the traffic outflow from link z (PCE/T).

A crucial point of Eq. (1) is the dynamics of link outflows. A possible approach to describe traffic outflow in a given network is described by the theory of urban fundamental diagram which was first proposed by [3]. The theory is called macroscopic fundamental diagram (MFD). This concept has widely been investigated during the past decades, e.g. [8], [2], [4], [5], [9]. By using the analogy of the MFD concept, the outflows $q_{w,z}$ and q_z can be defined by restricting the traffic network to link level. This practically means that each link has a dedicated MFD model. MFD assumes the following fundamental relationship:

$$q = \rho \cdot v, \quad (2)$$

where ρ denotes the traffic density and v is the space mean speed on a link. There are several formulas available in the literature for v [14]. In this papaer, one of the basic relationships is used for describing the speed of link z (called Pipes-Munjal model [10], which is practically a modified version of Greenshields' model):

$$v_z(\rho) = v_z^{free}\left[1 - \left(\frac{\rho_z}{\rho_z^{jam}}\right)^a\right], \quad (3)$$

where v_z^{free} represents the free-flow speed (i.e. no congestion), ρ_z^{jam} is the jam density (practically a 'bumper-to-bumper' case within the road link) and a is an empirical parameter. As traffic density is defined as

$$\rho_z = \frac{n_z}{l_z}, \quad (4)$$

(l_z is the link length) Eq. (3) can be recast as follows:

$$v_z(n_z) = v_z^{free}\left[1-\left(\frac{n_z}{n_z^{jam}}\right)^a\right]. \tag{5}$$

By substituting Eq. (5) into Eq. (2), the link-based traffic flow is derived:

$$q_z = \rho_z v_z = \frac{n_z}{l_z} v_z^{free}\left[1-\left(\frac{n_z}{n_z^{jam}}\right)^a\right]. \tag{6}$$

Note that flow q_w is also calculated by the formula of Eq. (6) concerning link w.

Two-fluid model for link-based traffic description

The two-fluid model [6] consideres as the whole traffic flow was composed by two flows: the flow of moving vehicles and the flow of vehicles stopped in traffic lanes (e.g. at red signal, in traffic jams, for freight delivery etc). The model defines the fraction of stopped vehicles as f^s, which can represent the ratio of the time while a floating car circulating in a network is stopped divided by its whole travel time:

$$f^s = \frac{T^s}{T}. \tag{7}$$

The two-fluid model states that f^s can be given in term of concentration:

$$f^s = \left(\frac{\rho}{\rho^{jam}}\right)^p, \tag{8}$$

where ρ^{jam} denotes the jam density and parameter p is the measure of quality of the traffic network. Substituting Eq. (4) into Eq. (8), f^s can be rewritten as:

$$f^s = \frac{T^s}{T} = \left(\frac{n}{n^{jam}}\right)^p. \tag{9}$$

The two-fluid model is usually applied to characterize a whole traffic network (town or districts). Nevertheless, the two-fluid approach is also valid for smaller networks. Therefore, a link-based two-fluid model can be given concerning link z as follows:

$$f_z^s = \frac{T_z^s}{T_z} = \left(\frac{n_z}{n_z^{jam}}\right)^p, \tag{10}$$

where T_z^s is the average stop time of the floating cars going through link z and T_z is the average travel time of vehicles on link z. Since f^s provides us information on queue lengths on links, it gives a more specific description of the traffic state on links than average travel time or speed would.

Model for link vehicle-count based on time-occupancy measurement

In road traffic technology the most common used sensor types are magnetic sensors and inductive loop-detectors. The time-occupancy parameter of these is calculated as follows:

$$o^t = \frac{\sum t^{occ}}{T}, \tag{11}$$

where $\sum t^{occ}$ denotes the sum of all occupancy times while the detector is covered by vehicles during sample time T. [11] derives the relationship between time-occupancy measurements of cross-sectional traffic detectors and the road link's space-occupancy. Space-occupancy is defined as the ratio of the sum of all vehicle lengths and the link length:

$$o_z^s = \frac{\sum l^{veh}}{l_z}. \tag{12}$$

Moreover, by considering a unit vehicle length l^{PCE}:

$$o_z^s = \frac{n_z \cdot l^{PCE}}{l_z}. \tag{13}$$

Time and space-occupancy values are quite similar [15], therefore the slight difference between them can be modeled by an appropriate noise term ζ:

$$o_z^t = o_z^s + \zeta = \frac{n_z \cdot l^{PCE}}{l_z} + \zeta. \tag{14}$$

Traffic modeling and measurement

The nonlinear traffic model

The discrete time state space representation of a nonlinear dynamics (without control input in this case) can be given by the following stochastic difference equation:

$$x(k+1) = f\left(x(k),\, \nu(k)\right), \tag{15}$$

with the measurement equation:

$$y(k) = g\left(x(k),\, \zeta(k)\right), \tag{16}$$

where $\nu(k)$ and $\zeta(k)$ represent the process and measurement noise respectively.
State vector is composed as follows:

$$x(k) = \begin{bmatrix} n_1(k) \\ n_2(k) \\ n_3(k) \\ \vdots \\ n_n(k) \end{bmatrix}, \tag{17}$$

where n_z denotes the number of vehicles on link z ($z = 1,\, 2, ...,\, n$).
Based on Eq. (1) the dynamics of each link z in Eq. (15) is given as

$$n_z(k+1) = n_z(k) + T\left[\sum_{w\in I_M} \alpha_{w,z} q_w(k) - q_z(k)\right] + \nu_z(k), \tag{18}$$

augmented by $\nu_z(k)$ as a noise term in the system.
Applying Eq. (6) for traffic flow dynamics, Eq. (18) finally becomes:

$$n_z(k+1) = n_z(k) + T\left[\sum_{w\in I_M} \alpha_{w,z}\frac{n_w(k)}{l_w} v_w^{free}\left[1-\left(\frac{n_w(k)}{n_w^{jam}}\right)^a\right] - \frac{n_z(k)}{l_z} v_z^{free}\left[1-\left(\frac{n_z(k)}{n_z^{jam}}\right)^a\right]\right] + \nu_z(k). \tag{19}$$

Sample time T can be long, even 15 minutes, therefore, the effect of signal controllers are taken into consideration as an average value which means that it is not necessary to know the signal programs.
According to the state space representation form, the measurement equation (16) must be defined as well. Using the models provided in the previous sections, the following measurements can be defined in the system:

- o_z^t is the time-occupancy on link z, measured by traffic detectors as given by Eq. (11).
- $f_z^s = \frac{T_z^s}{T_z}$ from Eq. (10) is detected as floating car data (FCD) for a single vehicle. Hence, the mean of all floating car measurements during the sample time can be calculated as

  $$\bar{f}_z^s = \frac{\sum_{i=1}^{num} f_{i,z}^s}{num}, \tag{20}$$

  where num denotes the number of cars measured on link z. As p is a constant parameter, Eq. (10) can be rearranged:

  $$\left(\bar{f}_z^s\right)^{1/p} = \frac{n_z}{n_z^{jam}}. \tag{21}$$

  Therefore $\left(\bar{f}^s\right)^{1/p}$ is considered as a measured value.

Finally, the discrete time measurement equation is given as follows:

$$\underbrace{\begin{bmatrix} o_1^t(k) \\ o_2^t(k) \\ \vdots \\ o_n^t(k) \\ \left(\bar{f}_1^s\right)^{1/p}(k) \\ \left(\bar{f}_2^s\right)^{1/p}(k) \\ \vdots \\ \left(\bar{f}_n^s\right)^{1/p}(k) \end{bmatrix}}_{y(k)} = \underbrace{\begin{bmatrix} \frac{l^{PCE}}{l_1} & & & \\ & \frac{l^{PCE}}{l_2} & & \\ & & \ddots & \\ & & & \frac{l^{PCE}}{l_n} \\ \frac{1}{n_1^{jam}} & & & \\ & \frac{1}{n_2^{jam}} & & \\ & & \ddots & \\ & & & \frac{1}{n_n^{jam}} \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} n_1(k) \\ n_2(k) \\ n_3(k) \\ \vdots \\ n_n(k) \end{bmatrix}}_{x(k)} + \underbrace{\begin{bmatrix} \zeta_1^{det}(k) \\ \zeta_2^{det}(k) \\ \zeta_3^{det}(k) \\ \vdots \\ \zeta_n^{det}(k) \\ \zeta_1^{FCD}(k) \\ \zeta_2^{FCD}(k) \\ \zeta_3^{FCD}(k) \\ \vdots \\ \zeta_n^{FCD}(k) \end{bmatrix}}_{\zeta(k)} \tag{22}$$

Linearization of the nonlinear traffic model

To better deal with the nonlinear dynamics given in the previous section, the linearization technique via Taylor series [13] can be used for Eqs. (15)-(16), i.e. the real state x and measurement y vectors are approximated:

$$x(k+1) \approx f(\hat{x}(k),0) + \frac{\partial f(\hat{x}(k),0)}{\partial x}\left(x(k) - \hat{x}(k)\right) + \frac{\partial f(\hat{x}(k),0)}{\partial \nu}\nu(k), \tag{23}$$

$$y(k) \approx g(\hat{x}(k),0) + \frac{\partial g(\hat{x}(k),0)}{\partial x}\left(x(k) - \hat{x}(k)\right) + \frac{\partial g(\hat{x}(k),0)}{\partial \zeta}\zeta(k), \tag{24}$$

where $\hat{x}(k)$ denotes the estimate of the state at discrete time step k.

Practically, the linearization means the calculation of Jacobian matrices of partial derivatives of functions (15)-(16):

$$\begin{aligned} A(k) = &\quad \tfrac{\partial f(\hat{x}(k),0)}{\partial x}, && (25)\\ B_\nu(k) = &\quad \tfrac{\partial f(\hat{x}(k),0)}{\partial \nu}, && (26)\\ C(k) = &\quad \tfrac{\partial g(\hat{x}(k),0)}{\partial x}, && (27)\\ C_\zeta(k) = &\quad \tfrac{\partial g(\hat{x}(k),0)}{\partial \zeta}. && (28) \end{aligned}$$

By using the simplified notation of (25)-(28) for Eqs.(23)-(24), the following formulas are obtained:

$$\begin{aligned} x(k+1) &\approx \tilde{x}(k) + A(k)\left(x(k) - \hat{x}(k)\right) + B_\nu(k)\nu(k), && (29)\\ y(k) &\approx \tilde{y}(k) + C(k)\left(x(k) - \hat{x}(k)\right) + C_\zeta(k)\zeta(k), && (30) \end{aligned}$$

where $\tilde{x}(k)$ and $\tilde{y}(k)$ are the approximated state and measurement variables.

Accordingly, the linearized matrices must be determined. The formula described by (25) is meant as differentiation by each element of state vector x. Therefore, for the state equation (19) two basic cases are given:

1. If the differentiation is done by state variable indexed by z (i.e. n_z):

$$\frac{\partial n_z(k+1)}{\partial n_z} = 1 - T\frac{v_z^{free}}{l_z}\left(1 - (a+1)\left(\frac{n_z(k)}{n_z^{jam}}\right)^a\right). \tag{31}$$

2. If the differentiation is done by state variable indexed by w (i.e. n_w):

$$\frac{\partial n_z(k+1)}{\partial n_w} = \alpha_{w,z}T\frac{v_w^{free}}{l_w}\left(1 - (a+1)\left(\frac{n_w(k)}{n_w^{jam}}\right)^a\right). \tag{32}$$

Jacobian matrix B_ν is resulted as

$$B_\nu(k) = I. \tag{33}$$

The Jacobian matrices of the measurement equation (22) are given as follows:

$$C = \begin{bmatrix} \frac{l^{PCE}}{l_1} & & & \\ & \frac{l^{PCE}}{l_2} & & \\ & & \ddots & \\ & & & \frac{l^{PCE}}{l_n} \\ \frac{1}{n_1^{jam}} & & & \\ & \frac{1}{n_2^{jam}} & & \\ & & \ddots & \\ & & & \frac{1}{n_n^{jam}} \end{bmatrix}, \tag{34}$$

$$C_\zeta(k) = I. \tag{35}$$

Robust state estimation

Since we have a system of which the noise descriptions (the statistical properties of turning rates) are unknown, Kalman/H_∞ filter is applied to resolve robust state estimation problem, according to the description provided in [12].

Uncertainty in the traffic model

Turning rate $\alpha_{w,z}$ is a quite ambiguous point of the traffic model described in Eq. (19). Obviously, one is able to estimate this term based on previous measurements. However, exact reliable values cannot be found for turning rates as they are strongly stochastic variables. Therefore, a robust approach can be applied for state estimation. By following the method of robust Kalman/H_∞ filtering for a linear system [12], uncertainties can be encapsulated into the linearized system model (29) derived previously:

$$x(k+1) \approx \tilde{x}(k) + (A(k) + \Delta A(k))\,(x(k) - \hat{x}(k)) + B_\nu(k)\nu(k), \tag{36}$$

where ΔA denotes the uncertainty matrix concerning the turning rates. The uncertainty matrix is assumed to be of the following structure:

$$\Delta A(k) = M(k)\Gamma(k)E(k), \tag{37}$$

where $M(k)$ and $E(k)$ are known real constant matrices of appropriate dimensions, and $\Gamma(k)$ is an unknown real time-varying matrix satisfying the following inequality:

$$\Gamma^T(k)\Gamma(k) \leq I. \tag{38}$$

Kalman/H_∞ filter design with data fusion

Apart from robust state estimation, the designed filter must also be able to fuse data collected from different sensor sources. These sensors can either be installed into the road infrastructure or can be in connection with the movement of vehicles, for example floating car data (FCD), or floating mobile data (FMD). On those links where there is at least one built-in road traffic sensor, data is generated continuously, therefore, the estimation of the Kalman Filter can always be updated, even if FCD is available. On links where no built-in detector is installed, the continuous Kalman Filter update cannot be guaranteed, since measurement data is only generated if there is a vehicle equipped with such device. If there is no measurement data in a period, the intermittent Kalman Filter technique is used, i.e. the state estimate of the previous time-step is simply propagated [7].

Example

A minimal example modelling a simple junction (see Fig.1) is provided to show how the proposed method can be applied. The state of the network is represented by the number of vehicles on links while traffic information is collected from on-street traffic detectors and moving vehicles.

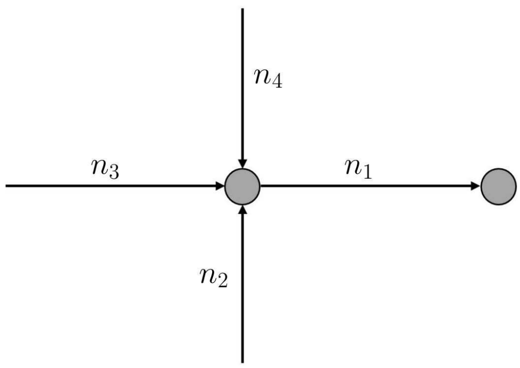

Figure 1: Example network

The discrete time system model is given as follows:

$$\begin{bmatrix} n_1(k+1) \\ n_2(k+1) \\ n_3(k+1) \\ n_4(k+1) \end{bmatrix} = \begin{bmatrix} n_1(k) + T\sum_{w=2}^{4} \alpha_{w,1} \frac{n_w(k)}{l_w} v_w^{free}\left[1 - \left(\frac{n_w(k)}{n_w^{jam}}\right)^a\right] - T\frac{n_1(k)}{l_1} v_1^{free}\left[1 - \left(\frac{n_1(k)}{n_1^{jam}}\right)^a\right] + \nu_1(k) \\ n_2(k) + d_2(k) - T\frac{n_2(k)}{l_2} v_2^{free}\left[1 - \left(\frac{n_2(k)}{n_2^{jam}}\right)^a\right] + \nu_2(k) \\ n_3(k) + d_3(k) - T\frac{n_3(k)}{l_3} v_3^{free}\left[1 - \left(\frac{n_3(k)}{n_3^{jam}}\right)^a\right] + \nu_3(k) \\ n_4(k) + d_4(k) - T\frac{n_4(k)}{l_4} v_4^{free}\left[1 - \left(\frac{n_4(k)}{n_4^{jam}}\right)^a\right] + \nu_4(k) \end{bmatrix}, \tag{39}$$

where $d_w(k)$ denotes vehicle input demand appearing at the boundary of the traffic network entering to link indexed by $w = 2,\ 3,\ 4$.

Assuming that traffic detector stations are only present on link 2 and 4 the discrete time measurement equation is given as follows:

$$\begin{bmatrix} o_2^t(k) \\ o_4^t(k) \\ \left(\bar{f}_1^s\right)^{1/p}(k) \\ \left(\bar{f}_2^s\right)^{1/p}(k) \\ \left(\bar{f}_3^s\right)^{1/p}(k) \\ \left(\bar{f}_4^s\right)^{1/p}(k) \end{bmatrix} = \begin{bmatrix} & \frac{l^{PCE}}{l_2} & & \\ & & & \frac{l^{PCE}}{l_4} \\ \frac{1}{n_1^{jam}} & & & \\ & \frac{1}{n_2^{jam}} & & \\ & & \frac{1}{n_3^{jam}} & \\ & & & \frac{1}{n_4^{jam}} \end{bmatrix} \begin{bmatrix} n_1(k) \\ n_2(k) \\ n_3(k) \\ n_4(k) \end{bmatrix} + \begin{bmatrix} \zeta_2(k)^{det} \\ \zeta_4(k)^{det} \\ \zeta_1(k)^{FCD} \\ \zeta_2(k)^{FCD} \\ \zeta_3(k)^{FCD} \\ \zeta_4(k)^{FCD} \end{bmatrix} \tag{40}$$

The linearization provides:

$$A(k) =$$
$$\begin{bmatrix} 1-\frac{Tv_1^{free}}{l_1}(1-\frac{(a+1)n_1^a(k)}{(n_1^{jam})^a}) & \alpha_{2,1}T\frac{v_2^{free}}{l_2}(1-\frac{(a+1)n_2^a(k)}{(n_2^{jam})^a}) & \alpha_{3,1}T\frac{v_3^{free}}{l_3}(1-\frac{(a+1)n_3^a(k)}{(n_3^{jam})^a}) & \alpha_{4,1}T\frac{v_4^{free}}{l_4}(1-\frac{(a+1)n_4^a(k)}{(n_4^{jam})^a}) \\ & 1-\frac{Tv_2^{free}}{l_2}(1-\frac{(a+1)n_2^a(k)}{(n_2^{jam})^a}) & & \\ & & 1-\frac{Tv_3^{free}}{l_3}(1-\frac{(a+1)n_3^a(k)}{(n_3^{jam})^a}) & \\ & & & 1-\frac{Tv_4^{free}}{l_4}(1-\frac{(a+1)n_4^a(k)}{(n_4^{jam})^a}) \end{bmatrix}. \tag{41}$$

$$B_\nu(k) = I. \tag{42}$$

The Jacobian matrices of the measurement equation (22) are given as follows:

$$C = \begin{bmatrix} & \frac{l^{PCE}}{l_2} & & \\ & & & \frac{l^{PCE}}{l_4} \\ \frac{1}{n_1^{jam}} & & & \\ & \frac{1}{n_2^{jam}} & & \\ & & \frac{1}{n_3^{jam}} & \\ & & & \frac{1}{n_4^{jam}} \end{bmatrix}, \tag{43}$$

$$C_\zeta(k) = I. \tag{44}$$

The next step is to determine the uncertainty matrix ΔA modeling the ambiguity of the turning rates. According to the formula of (37), $M(k)$ and $E(k)$ are defined as follows:

$$M(k) = \begin{bmatrix} 0 & T\frac{v_2^{free}}{l_2}(1-\frac{(a+1)n_2^a(k)}{(n_2^{jam})^a}) & T\frac{v_3^{free}}{l_3}(1-\frac{(a+1)n_3^a(k)}{(n_3^{jam})^a}) & T\frac{v_4^{free}}{l_4}(1-\frac{(a+1)n_4^a(k)}{(n_4^{jam})^a}) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{45}$$

$$E(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \delta_{2,1}\cdot\alpha_{2,1} & 0 & 0 \\ 0 & 0 & \delta_{3,1}\cdot\alpha_{3,1} & 0 \\ 0 & 0 & 0 & \delta_{4,1}\cdot\alpha_{4,1} \end{bmatrix}, \tag{46}$$

where $\delta_{2,1}, \delta_{3,1}$ and $\delta_{4,1}$ are uncertainty factors that weight turning rates. For example, $\delta = 0.1$ expresses that the applied nominal turning rates $\alpha_{w,z}$ of the model might vary by $\pm 10\%$.

Simulation

The operation of the filter is tested based on simulation data that were generated using PTV Vissim microscopic traffic simulation software. The network shown in Fig. 1 was implemented into PTV Vissim where the state of traffic was evaluated in 1 minute long periods. Occupancy data were collected from links 2 and 4 and two-fluid data were collected from each link. The filter estimates from these input data the number of vehicles on the links. The exact number of vehicles were also measured and were compared to the estimated number provided by the filter. The expected operation of the filter can be seen in Fig 2, even if the performance of the model does not reach this accuracy so far.

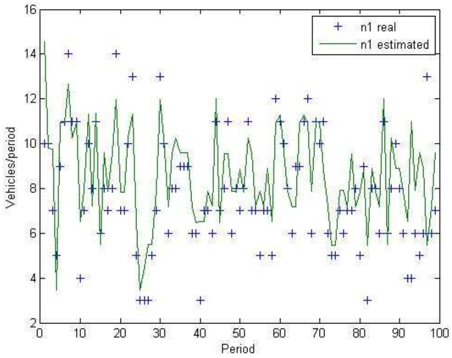

Figure 2: Expected result of simulation (n1 is the number of vehicles on link 1)

Conclusions

Intermittent data generated by vehicles and transport infrastructure is not reliable for estimating the state of the whole network. Therefore, the methodology presented in this paper uses both link-based and network-based macroscopic traffic models of which the results are combined by a data fusion technique using Kalman/H_∞ filter. The model uses a robust approach, therefore uncertainties in the traffic model (especially in turning rates) are treated as well, however, the performance of the simulation example needs to be developed.

References

[1] Csiszar, C. and Foldes, D. (2015) Analysis and Modelling Methods of Urban Integrated Information System of Transportation. *Smart Cities Symposium 2015, Prague*, ISBN 978-1-4673-6727-1

[2] Daganzo, C.F. and Geroliminis, N. (2008). An analytical approximation for macroscopic fundamental diagram of urban traffic. *Transportation Research Part B* 42(9): 771-781. http://dx.doi.org/10.1016/j.trb.2008.06.008

[3] Godfrey, J. (1969). The mechanism of a road network. *Traffic Engineering and Control* 11(7): 323-327.

[4] Geroliminis, N. and Daganzo, C.F. Existence of urban-scale macroscopic fundamental diagrams: Some experimental findings, in Transportation Research Part B: Methodological, vol. 42, num. 9, p. 759-770, 2008.

[5] Helbing, D. (2009). Derivation of a fundamental diagram for urban traffic flow. *The European Physical Journal B* 70(2): 229-241. http://dx.doi.org/10.1140/epjb/e2009-00093-7

[6] Herman, R. and Prigogine, I. (1979). A two-fluid approach to town traffic. *Science* 204(4389): 148-151., doi: 10.1126/science.204.4389.148

[7] Kar, S., Sinopoli, B., and Moura, J. M. F. (2012). Kalman Filtering With Intermittent Observations: Weak Convergence to a Stationary Distribution. IEEE Transactions on Automatic Control. 57(2): 405-420. doi: 10.1109/TAC.2011.2161834

[8] Mahmassani, H., Williams, J., and Herman, R. (1987). Performance of urban traffic networks. *10th International Symposium on Transportation and Traffic Theory* 1-20. Amsterdam, The Netherlands.

[9] Mazloumian, A. Geroliminis, N., and Helbing, D. (2010). The spatial variability of vehicle densities as determinant of urban network capacity, *Philosophical Transactions of the Royal Society A* 368(1928): 4627-4647. http://dx.doi.org/10.1098/rsta.2010.0099

[10] Pipes, L.A. (1967). Car-following models and the fundamental diagram of road traffic. Transportation Research, 1: 21-29.

[11] Papageorgiou, M. and Vigos, G. (2008). Relating time-occupancy measurements to space-occupancy and link vehicle-count. *Transportation Research Part C* 16(1): 1-17. http://dx.doi.org/10.1016/j.trc.2007.06.001

[12] Dan Simon. (2006). Optimal State Estimation: Kalman, H Infinity, and Nonlinear Approaches. Wiley-Interscience. ISBN:0471708585

[13] Stengel, R. F. (1987). Optimal Control and Estimation. Dover Publications, Series of Dover books on advanced mathematics, ISBN 9780486682006

[14] Wang, H., Li, J., Chen, Q.-Y., and Ni, D. (2009). Speed-density relationship: From deterministic to stochastic. *The 88th Transportation Research Board (TRB) Annual Meeting. Washington, DC*

[15] Zhang, W. and Rakha, H. (2005). Estimating traffic stream space-mean speed and reliability from dual and single loop detectors. Transportation Research Record, 1925: 38-47