Effect of periodic chip formation on the stability of turning processes

Gergely Gyebrószki*, Daniel Bachrathy*, Gábor Csernák** and Gabor Stepan*
*Department of Applied Mechanics,
Budapest University of Technology and Economics, Budapest, Hungary
**MTA-BME Research Group on Dynamics of Machines and Vehicles, Budapest, Hungary.

Summary. The analysis of chip formation and segmentation is characterized by hardships and problems, since even simple models could lead to complex thermodynamical and time-delay differential equations. We present a coupled system of the delay-differential equation of turning and the 4-dimensional chip-segmentation model of Csernák and Pálmai [1] and examine the stability properties of the resulting model. The multi-scale nature of the coupled system – namely that the time scale of the chip segmentation can be considerably smaller than the time scale of the turning – allows us to use averaging for the dynamics of the chip formation, thus enabling faster simulations. We point out, that in case of realistic parameters, the general effect of the chip segmentation is a sudden change in the bifurcation point.

The coupled system of turning and chip formation

Consider the delay differential equation of the turning:

$$\ddot{x}(\hat{t}) + 2\,\kappa\,\dot{x}(\hat{t}) + x(\hat{t}) = H_d(h_0 - x(\hat{t}) + x(\hat{t} - \tau)), \tag{1}$$

where $\hat{t} = \omega_n\,t$ is the dimensionless time, ω_n is the natural frequency, κ is the relative damping and h_0 is the chip thickness, Ω is the angular velocity and $\tau = 2\pi/\Omega$ is the delay. The 4D model of chip segmentation [1] has a different dimensionless time $\bar{t} = t/K$. Transforming the model to the dimensionless time of the turning, $\bar{t} = \hat{t}/(K\,\omega_n)$, we obtain:

$$\begin{aligned}
\dot{\tau}_2(\hat{t}) &= (1 - F_1 - F_2)/(K\,\omega_n) \\
\tau_1(\hat{t}) &= p\,\tau_2(\hat{t}) + s, \\
\dot{T}_0(\hat{t}) &= (\zeta(T_1(\hat{t}) - 2\,T_0(\hat{t})) - \xi\,T_0(\hat{t}))/\omega_n, \\
\dot{T}_1(\hat{t}) &= (\eta\,\tau_1(\hat{t})F_1 - \zeta(2\,T_1(\hat{t}) - T_2(\hat{t}) - T_0(\hat{t})) - \xi(T_1(\hat{t}) - T_0(\hat{t})))/\omega_n, \\
\dot{T}_2(\hat{t}) &= (\eta\tau_2(\hat{t})F_2 - (\xi + \zeta)(T_2(\hat{t}) - T_1(\hat{t})))/\omega_n
\end{aligned} \tag{2}$$

where ξ, ζ and η are system parameters depending on Ω, R and other thermodynamical parameters. The matching constitutive equation and thermodynamic time constant are:

$$F_i(\tau_i, T_i) = \frac{T_i + 1}{c + 1}\exp\frac{\tau_i - 1 + a(T_i - c)}{b(T_i + 1)} \qquad\qquad K = \frac{\tau_\Phi\,h(\hat{t})^2}{E\,L\,\Omega\,R\,\sin(\Phi)^2\,\cos(\Phi)^2}$$

Here a, b are material constants, c is the temperature of the shear zone, E is the modulus of elasticity and Φ is the angle of shear plane. To obtain a coupled model, substitute $h(\hat{t}) = h_0 - x(\hat{t}) + x(\hat{t} - \tau)$, introduce H_w as the product of constant coefficients and τ_2 on the right hand side of Eq. 1:

$$\ddot{x}(\hat{t}) + 2\,\kappa\,\dot{x}(\hat{t}) + x(\hat{t}) = H_w\,\tau_2(h(\hat{t}))\left(h_0 - x(\hat{t}) - x(\hat{t} - \tau)\right). \tag{3}$$

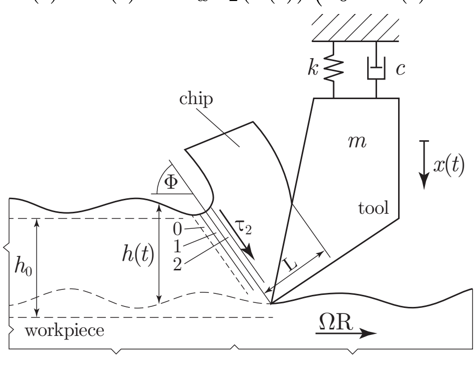

Figure 1: Model of shear zone, numbers indicate shear layers.

Averaging of the fast dynamics of chip formation

In case of realistic system parameters, the dynamics of the chip formation is 3–5 magnitudes faster, therefore it is reasonable to examine an approximation, where the fast dynamics is replaced by its average value. In Fig. 2.b), simulation for $h_0 = 0.018$ is shown, and in Fig. 2.a) the maximum, minimum and average of τ_2 is plotted for different h values based on numerical simulations. One can see in Fig. 2.a), that the average shear stress can be approximated as:

$$\tau_2(h) = \begin{cases} \overline{\tau}_2 & h \le h_{crit} \\ \alpha\,h^\beta + \gamma & h > h_{crit} \end{cases} \tag{4}$$

where α, β and γ are coefficients fitted to the simulated average of $\tau_2(h)$. The characteristic polynomial of the delay differential equation of turning in case of averaged τ_2 can be written as:

$$(i\,\omega)^2 + 2\,\kappa\,(i\,\omega) + 1 - H_w(-1 + e^{-(i\,\omega)\,\tau})\,(\tau_2(h) + h\,\dot{\tau}_2(h)) \tag{5}$$

The stability regions for Eq. (5) were constructed using multi-dimensional bisection method [3] for cases considering or neglecting the derivative of the average ($\dot{\tau}_2(h)$), see Fig. 2. Results show, that taking the derivative of the average into account increases the stable region.

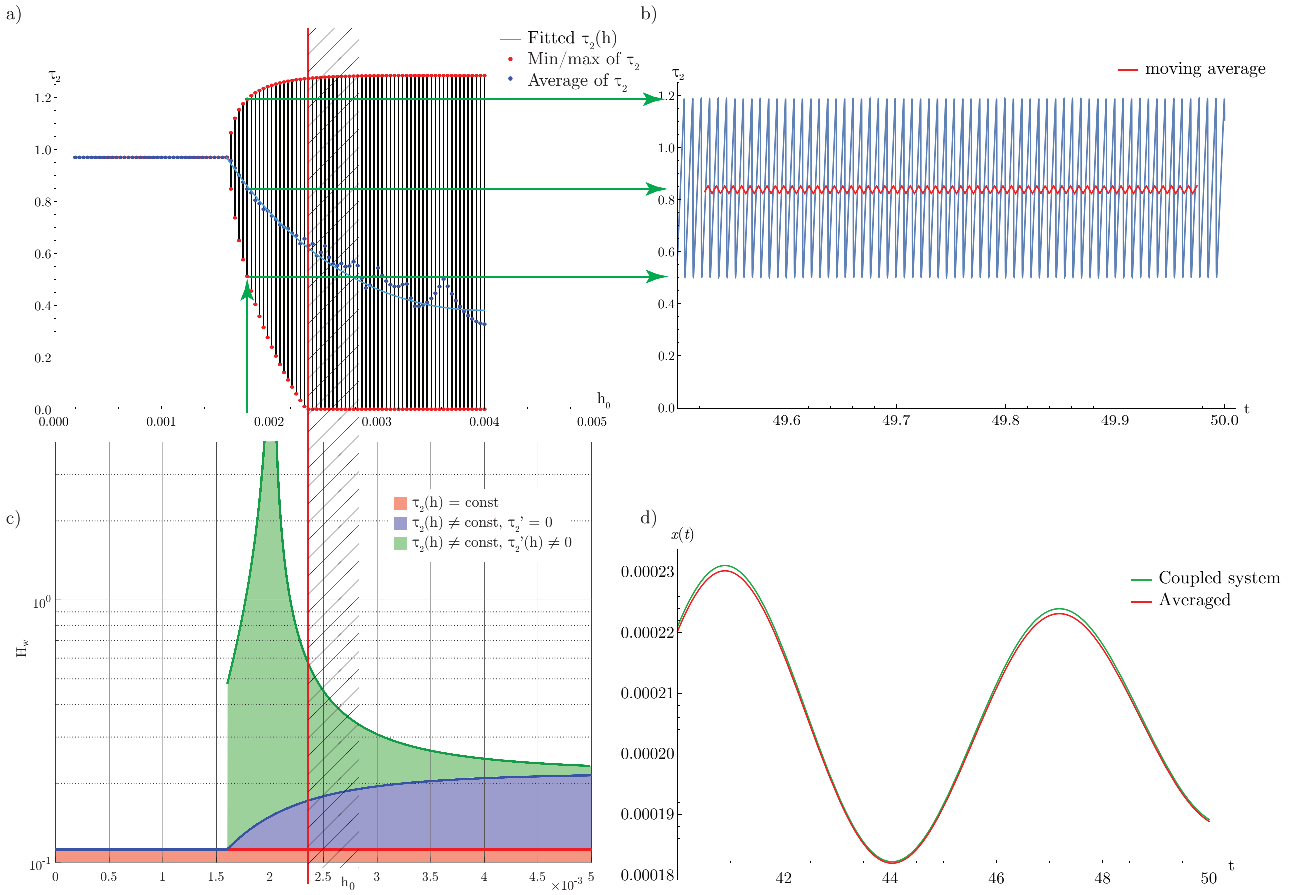

Figure 2: *a)* Maximum and minimum τ_2 values with respect to h_0, Blue dots indicate the average τ_2 values. The vertical red line indicates the critical h value, where chip segmentation begins. *b)* Example solution in case of $h_0 = 1.8 \times 10^{-3}$, red line indicates moving average of τ_2 *c)* Stability regions of the coupled system. Blue line indicates the case, where the derivative of the average with respect to h is neglected ($\tau_2(h) \equiv \overline{\tau}_2, \tau_2'(h) = 0$), green line indicates the case, when it is taken into account ($\tau_2'(h) \neq 0$). *d)* Comparison of the original and averaged solution.

Conclusions

The coupled model of chip formation and regenerative turning model is presented and its multi-scale nature is shown. An averaged solution is formulated and its stability is analysed. We found, that increasing the chip thickness decreases the average shear stress in the shear layer if oscillatory chip formation begins, and thus increases the stability, shifting the first bifurcation point upwards. The chip formation model, however, has a limited range of validity (until the shear stress reaches zero, see Fig 2.), therefore these cases need further investigation.

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