Analysis of underactuated dynamic locomotion systems using perturbation expansion - the twistcar toy example

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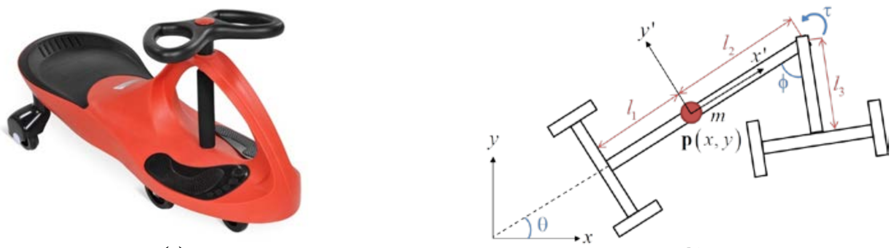
Summary. Underactuated robotic locomotion systems are commonly represented by nonholonomic constraints where in mixed systems, these constraints are also combined with momentum evolution equations. Such systems have been analyzed in the literature by exploiting symmetries and utilizing advanced geometric methods. These works typically assume that the shape variables are directly controlled, and obtain the system's solutions only via numerical integration. In this work, we demonstrate utilization of the perturbation expansion method for analyzing a model example of mixed locomotion system -- the twistcar toy vehicle, which is a variant of the well-studied roller racer model. The system is investigated by assuming small-amplitude oscillatory inputs of either steering angle (kinematic) or steering torque (mechanical), and explicit expansions for the system's solutions under both types of actuation are obtained. These expressions enable analyzing the dependence of the system's dynamic behavior on the vehicle's structural parameters and actuation type. In particular, we study the reversal in direction of motion under steering angle oscillations about the unfolded configuration, as well as influence of the choice of actuation type on convergence properties of the motion. Some of the findings are demonstrated qualitatively by reporting preliminary motion experiments with a modular robotic prototype of the vehicle.

Underactuated systems whose dynamics is governed by nonholonomic constraints have been a subject of extensive research for several decades [1-3]. While the earliest example of a nonholonomic system is the classical Chaplygin's sleigh [4], other typical examples of such systems are wheeled toy vehicles such as the snakeboard [5], roller racer [6-9] and more [10-13]. In the literature on robotic locomotion [14,15], many works have analyzed various locomotion systems subject to nonholonomic constraints, studying aspects such as nonlinear controllability and gait generation. One simple subclass of locomotion systems is *principal kinematic systems* [16], whose motion is time-invariant and depends only on geometric trajectories of shape variables (e.g. joint angles). On the other hand, the snakeboard and roller-racer examples belong to the more general class of *mixed systems*, where the motion is dynamic and governed also by momentum evolution in time. In a majority of previous works, it has been assumed that the controlled inputs are shape variables, which can be prescribed directly or via closed-loop feedback control in order to follow periodic trajectories (gaits). On the other hand, there are many practical cases where the controlled actuation is mechanical, i.e. forces or torques. Moreover, many systems do not employ closed-loop control due to practical limitations and apply open-loop oscillatory inputs instead [17].

In order to analyze locomotion systems under oscillatory inputs, Vela et al [18,19] developed an averaging theory for studying asymptotic solutions while assuming small-amplitude inputs. This theory utilizes separation of scales in the system's solution into fast oscillatory dynamics and slow 'averaged' dynamics. While this technique seems fairly general, it has been applied in [18,19] mainly for studying controllability and feedback stabilization of locomotion systems, where the controlled inputs were again limited to shape variables, i.e. kinematic actuation. Another asymptotic method which has recently been employed for locomotion dynamics, and to microswimmers in particular [20], is *perturbation expansion* [21]. The main advantage of this method is that it results in explicit expressions for the system's approximate solution under small-amplitude inputs, in contrast to other works in which solutions of the nonlinear equations of motion could only be obtained via numerical integration. The explicit expressions obtained from perturbation expansion enable analyzing dependence of the system's dynamic behavior on structural parameters and can also be utilized for optimizing the system's performance. To best of our knowledge, the perturbation expansion method has not yet been applied to mixed locomotion systems governed by nonholonomic constraints combined with momentum evolution.

The goal of this work (recently published in [22]) is to demonstrate the utility of perturbation expansion method for analysis of mixed locomotion systems, by providing a detailed investigation of a particular example problem -- the *twistcar*, which is a popular children's toy vehicle shown in Fig. 1(a). The twistcar has two axles of passive wheels and its only actuation is through cyclic oscillations of the steering handlebar, which makes it a highly underactuated system. Nonholonomic constraints are induced by the assumption that the wheels cannot slip sideways along the directions of their axles. We consider oscillatory inputs of either steering handlebar angle (kinematic) or the applied steering torque (mechanical). Perturbation expansion method is used in order to obtain explicit solutions under small amplitude approximation. It is shown that the cases of steering angle input and torque inputs differ fundamentally in terms of convergence or divergence of the vehicle's orientation angle. The latter case is further analyzed by obtaining the averaged solution which evolves on a slower time scale. Moreover, we show that changing the vehicle's structural parameters can have a drastic effect on the dynamics, and may even result in reversal of the direction of the vehicle's net motion.

The simple planar model of the twistcar, shown in Fig. 1(b), is in fact very similar to the roller-racer model, which has been studied extensively in the literature on nonholonomic mechanics [6-9]. The main difference between the two models is the fact that in the twistcar (TC), the relative angle of the steering link oscillates about $\phi=0$ whereas in the roller-racer (RR) the link is "unfolded" and oscillates about $\phi=\pi$. The works [6-9] have made two additional simplifying assumptions on the roller-racer model, which are questionable: first, they assumed that the steering link



(a) (b)

Fig. 1: (a) The twistcar toy vehicle, and (b) its planar two-link model.

has zero mass and nonzero moment of inertia, which is unphysical. Second, they assumed that the center-of-mass of the body link is located on the back axle (i.e. l_1=0 in Fig. 1(b)), while in reality this may result in tendency of the vehicle to tip over. Like many other works on robotic locomotion systems, [6-9] also assumed that the actuation input is the steering angle $\phi(t)$ rather than the applied steering torque $\tau(t)$. All these assumptions are relaxed in our present study. Furthermore, in order to make our analysis accessible to a broader audience of the robotics research community, we chose not to use advanced notions of geometric mechanics such as Lie groups and Riemannian geometry as in previous works. Instead, the results are presented using elementary terminology of linear algebra, vector calculus, and ordinary differential equations. We also report preliminary motion experiments on a robotic prototype, that qualitatively demonstrate some of the theoretical results.

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