Homoclinic Chaos near resonances in coupled SQUID

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Summary. An rf superconducting quantum interference device (SQUID) consists of a superconducting ring interrupted by a Josephson junction (JJ). The induced supercurrents around the ring are determined by the JJ through the celebrated Josephson relations. We study the dynamics of a pair of parametrically-driven coupled SQUIDs arranged in series. The drive is through the alternating critical current of the JJs. This system exhibits rich nonlinear behavior, including chaotic effects. We take advantage of the weak damping that characterizes these systems to perform a multiple-scales analysis and obtain amplitude equations, describing the slow dynamics of the system. This picture allows us to expose the existence of homoclinic orbits in the dynamics of the integrable part of the slow equations of motion. Using high-dimensional Melnikov theory, we are able to obtain explicit parameter values for which these orbits persist in the full system, consisting of both Hamiltonian and non-Hamiltonian perturbations, to form so called Silnikov orbits, indicating a loss of integrability and the existence of chaos.

Introduction

A superconducting quantum interference device (SQUID) is a device consisting of tiny loops of superconductors employing the so-called Josephson junction (i.e. a system made up of two superconductors, separated by a thin insulating layer [8]). SQUIDs can measure extremely weak signals due to their characteristic that the flux enclosed by the loops must be an integer number of flux quanta.
There are two main kinds of SQUIDs, the dc and the rf SQUIDs. The main difference of them is that the first has two Josepson junctions whereas the rf SQUID has only one [6]. When driven by an alternating magnetic field, the induced supercurrents around the ring are determined by the Josephson junctions through the celebrated Josephson relations. This system exhibits rich nonlinear behavior, including chaotic effects [5]. Quantum rf SQUIDs have attracted great attention, among others since they constitute essential elements for quantum computing [4]. Recently, rf SQUIDs have been shown to possibly serve as constituting elements for the so-called nonlinear magnetic metamaterials [1, 2].

Homoclinic chaos

Metamaterials are artificial, composite, inherently non-magnetic media with (positive or negative) magnetic response at microwave frequencies. Metamaterials provide access to all quadrants of the real permittivity-permeability plane, exhibiting negative refraction index, optical magnetism, and other fascinating properties. The key element for the construction of metamaterials has customarily been the split-ring resonator, which is a subwavelength resonant “particle" with operating frequencies up to the optical range [12]. Thus, the split-ring resonator is effectively a kind of an artificial “magnetic atom" [10]. A periodic arrangement of split-ring resonators in space forms a magnetic metamaterial that exhibits high frequency magnetism and negative permeability [7].
An rf SQUID itself is normally modelled by an electrical circuit containing an inductance L in series with an ideal Josephson element I_c shunted by a capacitor C and a resistor R. The parameter L represents the self-inductance of the ring with f being the flux through the inductor, the resistor R plays the role of the resistance of the ring the Josephson junction and the capacitor C models the capacitance across the Josephson junction. The dynamic equation for the flux threading a SQUID ring can be obtained by applying the Kirchhoff laws, i.e.

$$\ddot{f} + \gamma\dot{f} + f + \hat{\beta}\sin(2\pi f) = 0 \tag{1}$$

where $\gamma \equiv \frac{\omega_0}{C^2}\frac{L}{R}$ (damping constant), $\hat{\beta} \equiv \frac{I_c L}{\Phi_0 C}$ (SQUID parameter), and $\tau \equiv \omega_0 t$ (normalized time).
Now for two SQUIDs arranged in series, let us denote I_1 the supercurrent induced in the first SQUID and I_2 in the second. Then the differential equations for the flux of the first SQUID f_1 and of the second SQUID f_2 in the normalized form are given by [2]

$$\begin{aligned} \ddot{f}_1 + \gamma\dot{f}_1 + f_1 + \hat{\beta}\sin(2\pi f_1) - \lambda f_2 = 0 \\ \ddot{f}_2 + \gamma\dot{f}_2 + f_2 + \hat{\beta}\sin(2\pi f_2) - \lambda f_1 = 0. \end{aligned} \tag{2}$$

Here, we wish to study the homoclinic chaos near resonances in the coupled SQUIDs described by (2) when the nonlinearity coefficient $\hat{\beta}$ is varying periodically in time with a frequency close to the natural frequency of the system. In particular, we show that the unperturbed system has a homoclinic orbit in their collective dynamics by applying the geometrical method of singular perturbation theory near the resonances and Melnikov theory of near integrable Hamiltonian system to predict the chaotic behavior near resonances [13]. Moreover, we state the theorem for the existence of multi-homoclinic orbits near resonance following [13, 9].Compared to the previous works, our system has a so-called softening nonlinearity as opposed to stiffening one, i.e. the nonlinearity has the opposite sign.

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