Wave Propagation in Nonlinear Implicit Lattices

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Summary. We study the existence and uniqueness results of periodic and asymptotic travelling waves of nonlinear lattices in implicit form. In particular, we consider damped nonlinear lattices driven by external forces. The dissipative nonlinear lattices describe the magnetic metamaterials have also mechanical analogues. Localized solutions include solitary waves of permanent form and traveling breathers which appear time periodic in a system of reference moving at constant velocity are studied. We obtain a general criterion for spectral stability of multi-site breathers for a small coupling constant. For the metamaterial lattices, we focus on periodic traveling wave due to the presence of periodic force. We employ topological and variational methods to study the existence and the stability of periodic waves The existence and the stability of asymptotic waves are also computed and discussed numerically.

Introduction

In recent years there has been considerable interest in finding so-called intrinsic localized modes of nonlinear lattice equations in one spatial dimension. A particularly delicate question is whether such excitations can be made to move without shedding any radiation. The general answer is that they cannot, due to the presence of the so-called PeierlsâĂŞNabarro barrier, which comes about because of the loss of spatial translation symmetry, and the consequent existence of localized modes only for certain fixed locations on the lattice. For example, in the context of DNLS equations with pure cubic onsite nonlinearity, it is known that site centered localized modes are always stable and intersite localized modes are always unstable. These intersite modes are only stabilized in the continuum limit, therefore excluding the possibility of genuine traveling localized solitary waves as traveling waves would quickly become pinned to a lattice site. However, if we are not restricted to purely onsite cubic terms but are instead free to choose more general discretizations of the nonlinear term in the NLS equation then both intersite and onsite localized modes can be neutrally stable leading to the possibility of finding truly localized traveling waves.

Implicit Nonlinear Lattices

In this talk we will review some resent results on nonlinear implicit lattices. Discrete breathers in both one-dimensional and two-dimensional implicit lattices were approximated numerically in the limit of small coupling constant ϵ [5, 6]. Excitations of discrete breathers near the edge of a one-dimensional lattice created by a truncated array of nonlinear split-ring resonators were considered numerically in [7].

First, we consider spectral stability of multi-site discrete breathers in the limit of small coupling constant ϵ. This limit is referred usually as the *anti-continuum* limit and it has been considered before in the context of spectral stability of discrete breathers in the standard discrete Klein–Gordon equation. Our main result shows that the stability criterion for multi-site breathers in the discrete Klein–Gordon equation differs from the one derived in the standard discrete Klein–Gordon equation, [4].

In the second part, we consider a lattice equation modelling one-dimensional metamaterials formed by a discrete array of nonlinear resonators. We focus on periodic travelling waves due to the presence of a periodic force [1, 2, 3]. The existence and uniqueness results of periodic travelling waves of the system are presented. Our analytical results are found to be in good agreement with direct numerical computations.

References

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