Nonlinear damping types in wake oscillator model for vortex-induced vibrations of 2DOF rigid structure

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Summary. In this work, vortex-induced vibrations (or VIVs) of the 2DOF rigid structure are considered using wake oscillator approach. The ongoing study aims to investigate different types of nonlinear damping in fluid oscillators and their role in accuracy of VIV prediction. The model proposed in [1] is used as the base model. The goal is pursued by calibration of the model through constrained nonlinear minimization using published experimental data [2] and subsequent validation using published data from different set-ups. It is shown that certain combinations of oscillators are the most beneficial for a range of cases, for example, for low mass ratio.

Introduction

Vortex-induced vibrations are the result of interaction of a fluid flow and slender structure. It can be experienced in the water by risers, free spans of underwater pipelines, umbilicals, and in the air by antennas, suspension bridges, skyscrapers and not only. In subsea production operations, VIVs increase the probability of failure when frequency of perturbations in the fluid coincides with the natural frequency of the structure. If this synchronization happens, the lift and drag forces on the structure and corresponding displacements increase significantly. The aim of VIV modelling is to determine the conditions when this resonance state (lock-in) can be expected, and the amplitudes of oscillations which the structure can develop.

The wake oscillator approach is commonly used in VIV modelling. It allows to simplify simulation of the fluid domain and obtain the oscillations of dimensionless components of forces, supported by empirical coefficients. Van der Pol or Rayleigh equations are traditionally employed as a fluid equation in this approach. However, the question about the type of damping which is more beneficial for accuracy of VIV prediction haven't been studied in details. To fill this gap, authors conduct simulations with various types of damping using already developed wake oscillator model for 2DOF rigid structure [1] which is shown in Fig. 1 (a). Although, modelling of flexible structures is usually required in the practical applications, due to complexity of the problem, it is essential to explore first all the possibilities to enhance quality of prediction for rigid structure before applying them to a more complex object. Thus the chosen model [1] is modified using various damping types. Then the set of created models is calibrated using experimental data [2] and validated to build the overall rating of oscillators in terms of accuracy of prediction.

Mathematical modelling

For the considered model, the equations of motion in dimensionless form (based on the formulation proposed in [1]) are given as:

$$\begin{aligned}
\ddot{X} + 2\zeta\dot{X} + \omega_{st}^2 X &= \frac{a}{2\pi St}(\Omega_R - KSt\omega_{st})^2 + \frac{b}{4\pi St}(\Omega_R - KSt\omega_{st})^2 w - \\
&- 2a(\Omega_R - KSt\omega_{st})\dot{X} + \frac{c}{2}(\Omega_R - KSt\omega_{st})q\dot{Y} + a\pi St\dot{Y}\dot{Y} + \\
&+ 2a\pi St\dot{X}\dot{X} - b(\Omega_R - KSt\omega_{st})w\dot{X}; \qquad (1)\\
\ddot{Y} + 2\zeta\dot{Y} + \omega_{st}^2 Y &= \frac{c(\Omega_R - KSt\omega_{st})^2}{4\pi St}q - a(\Omega_R - KSt\omega_{st})\dot{Y} + 2a\pi St\dot{X}\dot{Y} - \\
&- \frac{b}{2}w\dot{Y}(\Omega_R - KSt\omega_{st}) - cq\dot{X}(\Omega_R - KSt\omega_{st}); \qquad (2)
\end{aligned}$$

$$\begin{aligned}
\ddot{w} - 2\varepsilon_x(\Omega_R - KSt\omega_{st})\dot{w} + 2\varepsilon_x(\Omega_R - KSt\omega_{st})w^2\dot{w} + 4(\Omega_R - KSt\omega_{st})^2 w &= A_x\ddot{X}; \qquad (3)\\
\ddot{q} - \varepsilon_y(\Omega_R - KSt\omega_{st})\dot{q} + \frac{\varepsilon_y}{(\Omega_R - KSt\omega_{st})}\dot{q}^3 + (\Omega_R - KSt\omega_{st})^2 q &= A_y\ddot{Y}, \qquad (4)
\end{aligned}$$

where X, Y - displacements in in-line and cross-flow directions respectively; w, q - wake coefficients of drag and lift force respectively; ζ - damping ratio; $\varepsilon_x, \varepsilon_y$ - damping coefficients in fluid equations; Ω_R - vortex shedding frequency; K - lock-in delay coefficient; St - Strouhal number; ω_{st} - natural frequency of the structure; A_x, A_y - coupling coefficients; a, b, c - parameters depending on initial drag (C_{D0}), fluctuating drag (C_{D0}^{fl}), and lift (C_{L0}) coefficients as: $a = \frac{C_{D0}\rho_f D^2}{\pi St 4m_*}$; $b = \frac{C_{D0}^{fl}\rho_f D^2}{\pi St 4m_*}$; $c = \frac{C_{L0}\rho_f D^2}{\pi St 4m_*}$, where m_* - mass per unit length, ρ_f - density of the outer fluid, D - diameter of the cylinder. The lock-in delay coefficient K is introduced into the model as an actual difference between the reduced velocity values where the synchronization state begins in experiment and in theory. It can be interpreted as a measure of inertia of the real resonance state relatively to the theoretical one.

In this research, modifications of the fluid equations given above for w and q are considered by varying the damping terms. For example, the following pairs of the wake equations are investigated: Van der Pol for in-line and Van der Pol for cross-flow wakes (as in original model); Rayleigh for in-line and Van der Pol for cross-flow wakes, etc. Our study shows that one of the most successful options is the combination of Van der Pol for in-line and Rayleigh for cross-flow wakes. This modification provides the advantageous fit for cases with low mass ratios $\mu \in (2, 4)$. Combinations of traditionally applied Van der Pol and Rayleigh damping terms with more rare Krenk-Nielsen and Landl damping terms are also considered as viable alternatives in this research on the basis of delivered accuracy.

Calibration and results for mass ratio 2.36

Optimisation of dimensionless parameters is conducted for each modification using constrained nonlinear minimization in Matlab Optimization Toolbox. The fit criteria for resonance curve ranged from the most significant to the least significant in the following order: 1) maximum developed amplitude, 2) amplitudes near highest peak, 3) point where lock-in begins, and end point of the super-upper branch, 4) amplitudes in the points equally distributed on the initial and lower branches. The optimization is performed for a minimum of 9 parameters for each modification.
The fit provided by the combination of Van der Pol for in-line and Rayleigh for cross-flow wakes is one of the best options identified in this research for mass ratio 2.36. Accuracy of this option is illustrated in Fig. 1 (b, c). Mass ratio in this research is defined as $\mu = \frac{m_s}{m_f}$, where m_s - mass of the considered structure, m_f - mass of the displaced fluid. The calibrated coefficients are: $C_{L0} = 1.87$; $C_{D0} = 2.30$; $C_{D0}^{fl} = 0.47$; $\varepsilon_x = 2.1276$; $\varepsilon_y = 0.064447$; $A_x = 12.24$; $A_y = 3.78$; $C_a = 4.06$ (fluid added mass coefficient); $K = 0.94$. Validation using different experimental data confirms that the identified set of coefficients provides a decent accuracy of prediction for mass ratios $\mu \in (2, 4)$.

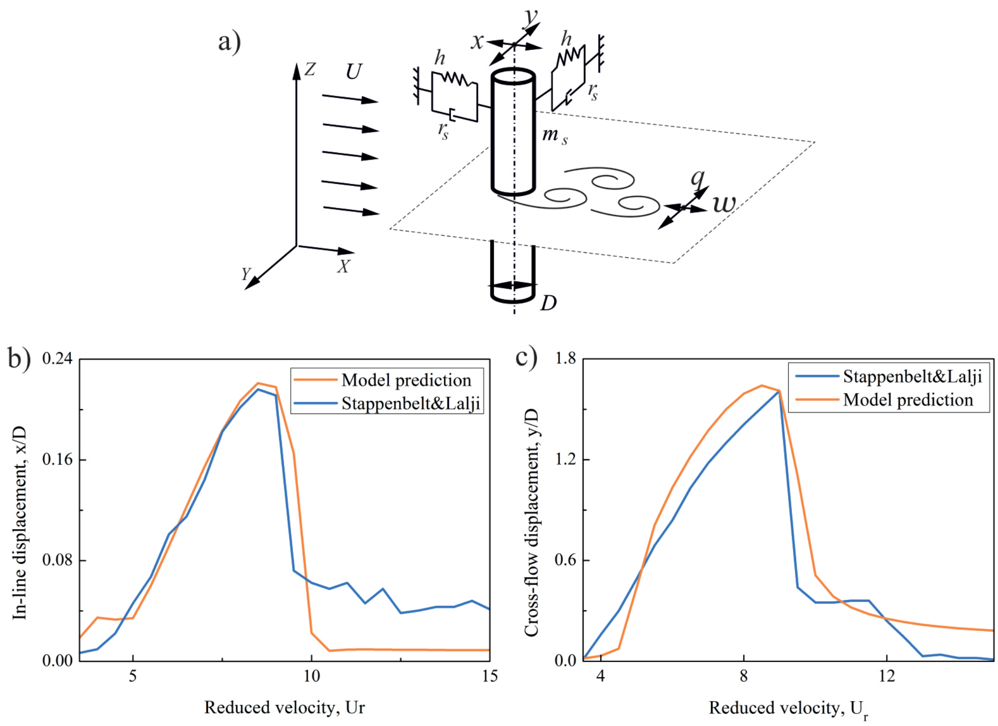

Figure 1: a) rigid elastically-supported structure with 2 degrees-of-freedom experiencing VIV; comparison of displacement predicted by the model and experimental data [2] for mass ratio 2.36 b) for in-line direction; c) for cross-flow direction.

Conclusions and future work

In this work, authors explore how different nonlinear damping terms in wake oscillator equation can influence the results of VIV modelling. The best fit options are identified for low mass ratio cases using alternative optimization procedures. Systematic validation with published data from different experimental set-ups confirms that identified best fit options are applicable for the mass ratio range $\mu \in (2, 4)$. The related study of the influence of damping terms on the accuracy of VIV prediction for medium $\mu \in (4, 7)$ and high $\mu \in (8, 13)$ mass ratios has to be conducted in the nearest future.

References

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