# Tweezer control for chimera states in small networks

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Summary. We propose a control scheme which can stabilize and fix the position of chimera states in small networks. Chimeras consist of coexisting domains of spatially coherent and incoherent dynamics in systems of nonlocally coupled identical oscillators. Chimera states are generally difficult to observe in small networks due to their short lifetime and erratic drifting of the spatial position of the incoherent domain. The control scheme, like a tweezer, might be useful in experiments, where usually only small networks can be realized.

Chimera states are complex spatio-temporal patterns in networks of coupled oscillators, exhibiting a hybrid nature of spatially coexisting coherent and incoherent domains. In small-size networks, there exist two difficulties preventing the observation of chimera states. First, it is known that chimera states are usually chaotic transients that eventually collapse to the uniformly synchronized state [1, 2]. Their mean lifetime decreases rapidly with decreasing system size such that one hardly observes chimeras for $20-30$ coupled oscillators. Second, the position of the incoherent domain is not stationary and moves erratically along the oscillator array [3, 4]. Both effects, finite lifetime and random walk of the chimera position, are negligible in large-size systems, but they dominate the dynamics of small-size systems, making the observation of chimera states very difficult.

We propose an efficient feedback control scheme which aims to stabilize chimera states in small networks [5]. Like a tweezer, which helps to hold tiny objects, our control has two levers: the first one prevents the chimera collapse, whereas the second one stabilizes its lateral position. Our control strategy is universal and effective for large as well as for small networks, it effectively works for oscillators exhibiting both phase and amplitude dynamics.

We consider a system of $N$ identical nonlocally coupled Van der Pol oscillators $x_k \in \mathbb{R}$ given by

$$\ddot{x}_k = (\varepsilon - x_k^2)\dot{x}_k - x_k + \frac{1}{R}\sum_{j=1}^{R}\left[a_-(x_{k-j}-x_k) + b_-(\dot{x}_{k-j}-\dot{x}_k)\right] + \frac{1}{R}\sum_{j=1}^{R}\Big[a_+(x_{k+j}-x_k) + b_+(\dot{x}_{k+j}-\dot{x}_k)\Big].$$

The scalar parameter $\varepsilon > 0$ determines the internal dynamics of all individual elements. For small $\varepsilon$ the oscillation of the single element is sinusoidal, while for large $\varepsilon$ it is a strongly nonlinear relaxation oscillation. Each element is coupled with $R$ nearest neighbors to the left and to the right. The oscillators are arranged on a ring (periodic boundary conditions) such that all indices are modulo $N$. The coupling constants in position and velocity to the left and to the right are denoted as $a_-$, $a_+$ and $b_-$, $b_+$, respectively. For the sake of simplicity we assume $a_- = a_+ = a,\ \ b_- = a\sigma_-,\ \ b_+ = a\sigma_+$.

We define two complex order parameters

$$Z_1(t) = \frac{1}{[N/2]}\sum_{k=1}^{[N/2]} e^{i\phi_k(t)}, \quad Z_2(t) = \frac{1}{[N/2]}\sum_{k=1}^{[N/2]} e^{i\phi_{N-k+1}(t)},$$

where $\phi_k(t)$ is the geometric phase of the $k$-th oscillator computed from $e^{i\phi_k(t)} = \left(x_k^2(t) + \dot{x}_k^2(t)\right)^{-1/2}\left(x_k(t) + i\dot{x}_k(t)\right)$.

Tweezer feedback control is introduced in the form

$$\sigma_\pm = K_{\rm s}\left(1 - \frac{1}{2}|Z_1 + Z_2|\right) \pm K_{\rm a}(|Z_1| - |Z_2|),$$

where $K_{\rm s}$ and $K_{\rm a}$ are gain constants for the symmetric and asymmetric parts of the feedback control, respectively.

We demonstrate the effect of the control scheme in small systems of nonlocally coupled Van der Pol oscillators, and investigate the role of system parameters and control strengths for the most efficient stabilization of chimera states. We show that tweezer control works successfully for Van der Pol oscillator in the weakly as well as in the strongly nonlinear regime (large $\varepsilon$), and for networks of nonlocally coupled FitzHugh-Nagumo oscillators [5].

## References

[1] Wolfrum M. and Omel'chenko O. E. (2011) Chimera states are chaotic transients. *Phys. Rev. E* **84**: 015201(R).

[2] Sieber J., Omel'chenko O. E., and Wolfrum M. (2014) Controlling Unstable Chaos: Stabilizing Chimera States by Feedback. *Phys. Rev. Letters* **112**: 054102.

[3] Omel'chenko O. E., Wolfrum M., and Maistrenko Yu. L. (2010) Chimera states as chaotic spatiotemporal patterns. *Phys. Rev. E* **81**: 065201(R).

[4] Bick C. and Martens E. (2015) Controlling Chimeras. *New J. Phys.* **17**: 033030.

[5] Omelchenko I., Omel'chenko O. E., Zakharova A., Wolfrum M., and Schöll E. (2016) Tweezers for Chimeras in Small Networks. *Phys. Rev. Letters* **116**: 114101.