Slow-invariant-manifold resonance capture in vortex-induced vibration of a circular cylinder with a nonlinear dissipative rotator

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Summary. At the intermediate Reynolds number $Re = 100$, a two-dimensional, linearly-sprung, circular cylinder restricted to move in the cross-flow direction and incorporating a rotational nonlinear energy sink (NES; consisting of a mass freely rotating about the cylinder axis and whose angular motion is restrained by a linear viscous damper) can undergo repetitive cycles of slowly decaying oscillations — which can lead to significant vortex street elongation with partial stabilization of the wake — interrupted by chaotic bursts. We construct a reduced-order model of the fluid–structure interaction dynamics and employ analytical techniques to show that the strongly modulated response is the manifestation of a resonance capture into a slow invariant manifold (SIM) that leads to targeted energy transfer from the cylinder to the rotator. Capture into the SIM corresponds to transient cylinder stabilization, and escape from the SIM to chaotic bursts.

Formulation of the fluid–structure interaction problem

We consider a two-dimensional circular cylinder of mass $M_{\rm cyl}$ and diameter D, immersed in an incompressible fluid of density $\rho_{\rm f}$ and kinematic viscosity ν, mounted on a linear spring of stiffness $K_{\rm cyl}$, and allowed to move only in the direction transverse to the free stream. A mass $M_{\rm NES}$ is attached to the cylinder and freely rotates at constant radius r_0 about the cylinder's generatrix (*cf.* Fig. 1). Inertial coupling allows energy to be transferred from the cylinder to the rotator, part of which is dissipated by a linear rotational viscous damper about the axis of rotation of the NES [1]. The Reynolds number $Re = UD/\nu$ is based on the cylinder diameter and on the free-stream velocity. We define a dimensionless time $\tau = tU/D$, scale the length, velocity and pressure by D, U and $\rho_{\rm f}U^2/2$, respectively, and introduce the dimensionless quantities $y = Y/D$, $\bar{r}_0 = r_0/D$, $m^* = \rho_{\rm b}/\rho_{\rm f}$ as well as a mass ratio $\epsilon_{\rm r} = M_{\rm NES}/(M_{\rm cyl} + M_{\rm NES})$. The equations governing the motion of the two coupled oscillators are then expressed in non-dimensional form as

$$\ddot{y} + {\omega_{\rm r}^*}^2 y = \epsilon_{\rm r}\bar{r}_0 \frac{\rm d}{{\rm d}\tau}(\dot{\theta}\sin\theta) + \frac{2C_{\rm L}}{\pi m^*} \quad \text{and} \quad \ddot{\theta} + \lambda_{\rm r}\dot{\theta} = \frac{\ddot{y}}{\bar{r}_0}\sin\theta, \tag{1}$$

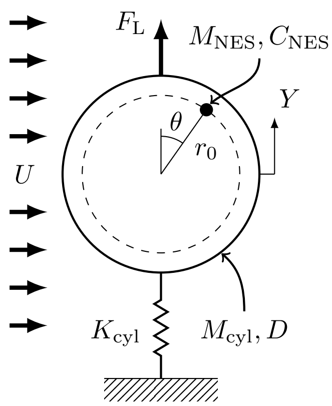

Figure 1: Circular cylinder in cross-flow with rotational NES.

where $\dot{(\,)} = {\rm d}(\,)/{\rm d}\tau$ and $\rho_{\rm b}$ is the density of the cylinder with the NES [2]. The lift force $F_{\rm L}$ has been scaled in favor of a dimensionless lift coefficient $C_{\rm L} = 2F_{\rm L}/(\rho_{\rm f}U^2 D)$, while the natural frequency of the linear spring supporting the cylinder has been non-dimensionalized as ${\omega_{\rm r}^*}^2 = (2\pi f_{\rm n}^*)^2 = D^2 K_{\rm cyl}/[U^2(M_{\rm cyl}+M_{\rm NES})]$, and a dimensionless damping coefficient has been defined as $\lambda_{\rm r} = DC_{\rm NES}/(UM_{\rm NES})$. The density ratio m^* is set equal to 10, and the Reynolds number to $Re = 100$. The natural frequency $f_{\rm n}^*$ of the linear spring is chosen close to the normalized shedding frequency of a stationary circular cylinder at $Re = 100$, which is equal to the Strouhal number $St = 0.167$. The cylinder is initially at rest, while the NES initial conditions are selected as $\theta(0) = \pi/2$ and $\dot{\theta}(0) = 0$. Our computational approach is based on the spectral-element code Nek5000 [3]. The Navier-Stokes equations are cast in an arbitrary Eulerian-Lagrangian frame and integrated forward in time in a staggered fashion, together with (1) governing the motion of the two coupled oscillators.

Physical phenomena for finite mass ratios

For NES parameters $\epsilon_{\rm r} = 0.33$, $\lambda_{\rm r} = 0.002745$ and $\bar{r}_0 = 0.458$, the dynamics is characterized by successive cycles of regular motion of the cylinder and the NES interspersed by chaotic bursts. During periods of regular motion, the cylinder oscillation amplitude gradually decreases and the NES becomes locked in a regime of steady rotation that corresponds to 1:1 resonance capture with the cylinder. As the slowly decaying cycle progresses, the amplitudes of the lift and drag coefficients reduce drastically, while the cylinder, NES and lift coefficient are in clear 1:1:1 resonance, dominated by a single frequency that slowly decreases and reaches a minimum right before the dynamics transitions toward instability. More interesting, as the lift coefficient approaches zero and the drag coefficient tends to a minimum, the structure of the flow aft of the cylinder changes noticeably. The attached vorticity is considerably elongated and straightened, suggesting that the steady, symmetric solution (unstable at $Re = 100$) is partially stabilized due to the indirect action of the NES on the flow (*cf.* Fig. 2) [2, 4].

Order reduction of the dynamics and numerical validation

We analytically study the dynamics of the system governed by (1) in the aforementioned regime of resonance [4]. The cylinder interacts with the surrounding fluid *only* through the lift coefficient C_{L}, for which numerical data provides enough information to formulate an *ansatz* $\hat{C}_{\mathrm{L}}$ for the asymptotic analysis. In order to apply classical complexification-averaging techniques, we impose the condition of 1:1:1 resonance and formally introduce the slowly varying instantaneous frequency $\omega = \omega(\tau)$ that depends on the amplitude of cylinder oscillations. We then use a slow-fast partition of the dynamics by letting $R(\tau)\mathrm{e}^{\mathrm{j}\zeta(\tau)} = \dot{y} + \mathrm{j}\omega(\tau)y$ and $\theta = \pm\zeta(\tau) + \psi(\tau)$. The instantaneous frequency ω and "fast" phase ζ are related through $\dot{\zeta}(\tau) = \omega(\tau)$. We assume an *ansatz* lift coefficient of the form $\hat{C}_{\mathrm{L}} = B(\tau)\sin[\zeta(\tau) + \eta(\tau)]$. In the above expressions, R, ψ, B and η are slowly varying functions of time. These steps are consistent with the numerical results in that they incorporate the essential assumption that, in the regime of 1:1:1 resonance, the NES, the cylinder and the lift coefficient are dominated by a single, slowly varying frequency ω and, consequently, that additional small-amplitude harmonics may be neglected.

Traditional averaging procedure yields the complex slow-flow equations, from which we extract the expression of the slow invariant manifold (SIM) of the problem, given by $R\cos\psi = -2\lambda_{\mathrm{r}}\bar{r}_0$. The SIM is composed of two branches (the one stable, the other unstable), and breaks down by the mechanism of saddle-node bifurcation. Past this point, the slow flow leaves the regime of 1:1:1 resonance and enters a different regime of dynamics. At the super-slow scale, on the stable branch of the SIM, further approximations and algebraic manipulations allow us to reduce the dynamics down to a system of one nonlinear ordinary differential equation governing the slow modulation of the fast oscillations of the cylinder during the slowly decaying cycle, $\dot{R} = f(R, \omega)$; and one algebraic equation relating the frequency of the cylinder oscillation to its amplitude, $\omega = g(R)$. The asymptotic analysis is validated by comparing the analytical predictions to numerical data obtained with Nek5000. Figure 2 shows the "sliding" of the trajectories on the stable branch of the slow invariant manifold. In the absence of a stable fixed point on the SIM, the "sliding" of the dynamics continues until the trajectory escapes the SIM and transitions into intermittent chaos, after which the dynamics is again captured on the stable branch of the SIM, and the previous cycle starts anew.

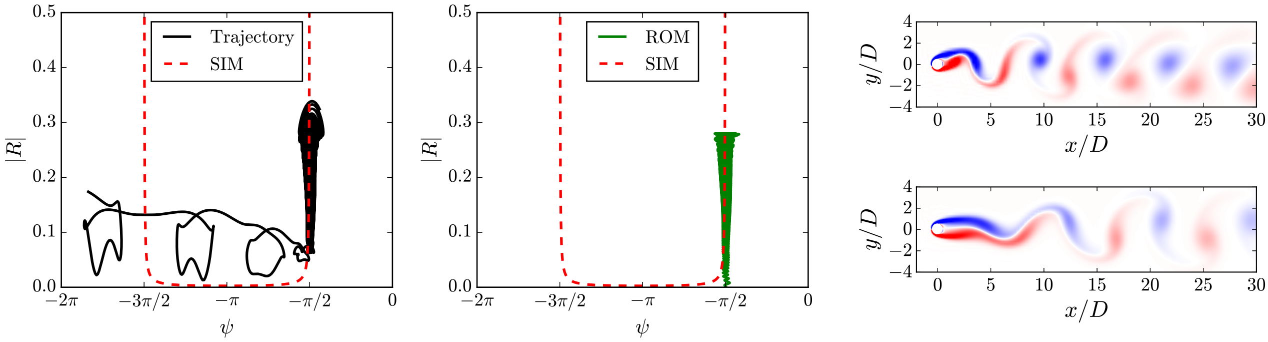

Figure 2: Capture into the 1:1 slow invariant manifold, for NES parameters $\epsilon_{\mathrm{r}} = 0.33$, $\bar{r}_0 = 0.458$ and $\lambda_{\mathrm{r}} = 0.002745$. From left to right: comparison between the raw data obtained by direct numerical simulation and the analytical prediction resulting from integration of the reduced-order slow-flow model (ROM); and spanwise vorticity field at the beginning (top right) and end (bottom right) of the slowly decaying cycle highlighting significant elongation of the attached vorticity.

Conclusions

The mechanism of passive VIV suppression of a sprung cylinder in cross-flow with a rotational NES characterized by successive cycles of regular, slowly decaying motion interrupted by chaotic bursts, is proven to be the result of a resonance capture into the stable branch of a slow invariant manifold. This clarifies the action of the rotator on the resonance dynamics of the fluid–structure interaction problem, including the partial wake stabilization observed for finite mass ratios.

References

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