Three-dimensional Energy Channeling in Unit-cell Model Coupled to a Spherical Rotator

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Summary. We study the complex nonlinear mechanism of uni- and bi-directional energy channeling realized in a locally resonant three-dimensional (3D) unit-cell model comprising of an external mass subjected to a 3D nonlinear local potential with a spherical internal rotator in the limit of low energy excitation. We discuss two families of non-stationary regimes corresponding to in-plane and out-of-plane energy channeling which are manifested by 3D transformation of general in-plane oscillations of the external element to orthogonally reoriented in-plane and out-of-plane oscillations. The energy flow is fully controlled by the orientation of the internal rotator. Numerical simulations are in very good correspondence with the multi scale analysis.

Introduction

The emergence of the resonant energy transfer mechanism has generated considerable interest in various aspects of modern physics and engineering such as nonlinear vibrations and waves in mechanical structures, physics of fluids, plasma physics, dynamics of nonlinear lattices and related fields. However, it requires theoretical understanding of the intrinsic mechanisms to predictively trigger the energy flow. Unlike the case of stationary or weakly non-stationary processes, analytical study of the highly non-stationary regimes is complicated. The conventional analytical methods such as nonlinear normal modes (NNMs) for studying stationary regimes can be inappropriate for studying non-stationary regimes of intense, resonant energy flow. These regimes can be excited in various finite as well as infinite, anharmonic oscillatory structures subjected to impulsive, periodic, quasi-periodic and random excitation. The concept of limiting phase trajectories (LPTs) was introduced by Manevitch [1], which is quite natural for a comprehensive description of the regime of intense energy transfer emerging in driven and coupled anharmonic oscillators, oscillatory chains as well as the oscillatory models with time varying parameters.

The study by Sigalov et al. [2] dwells on the interplay between regular and chaotic dynamical behavior exhibited by nonlinear inertial coupled linear oscillator and an eccentric rotator. Recent works by Vorotnikov et al. [3, 4] on the internal rotator inertially coupled to a primary structure in a 2D elastic potential considered the analysis of the formation and bifurcation of highly non-stationary regimes. The studies have considered the mechanism of uni- and bi-directional energy flow governed by the intrinsic resonant interactions in the limit of low and high energy excitations. Present study is devoted to the analysis of non-stationary regimes of 3D in-plane and out-of-plane, uni- and bi-directional energy channeling emerging in locally resonant 3D unit-cell model. This phenomenon is manifested by a complete resonant energy flow from arbitrarily oriented in-plane oscillations to orthogonal in-plane as well as out-of-plane oscillations of the external mass. This 3D energy flow is fully controlled by the motion of the internal spherical rotator. Using a regular multiple time scale analysis we derive the slow-flow model describing the evolution of amplitudes and phases of the three dimensional vibrations of the external mass and the internal rotator.

Mathematical Model and Results

The considered system of locally resonant external mass (of mass unity) oscillating in a 3D weakly nonlinear ($O(\varepsilon)$) local potential coupled to an internal rotator (of mass ε concentrated at unit radius) is as shown in Fig. 1. The non-dimensional equations of motion are,

$$\ddot{x} - \varepsilon \sin(\psi)\,\ddot{\psi} = -x - \varepsilon\alpha_1 x^3 + \varepsilon\cos(\psi)\,\dot{\psi}^2 \qquad (1a)$$

$$\ddot{y} - \varepsilon\sin(\psi)\sin(\theta)\,\ddot{\theta} + \varepsilon\cos(\psi)\cos(\theta)\,\ddot{\psi} = -y - \varepsilon\alpha_2 y^3 + \varepsilon\sin(\psi)\cos(\theta)\left(\dot{\theta}^2 + \dot{\psi}^2\right) + 2\varepsilon\cos(\psi)\sin(\theta)\,\dot{\theta}\dot{\psi} \qquad (1b)$$

$$\ddot{z} + \varepsilon\sin(\psi)\cos(\theta)\,\ddot{\theta} + \varepsilon\cos(\psi)\sin(\theta)\,\ddot{\psi} = -z - \varepsilon\alpha_3 z^3 + \varepsilon\sin(\psi)\sin(\theta)\left(\dot{\theta}^2 + \dot{\psi}^2\right) - 2\varepsilon\cos(\psi)\cos(\theta)\,\dot{\theta}\dot{\psi} \qquad (1c)$$

$$-\sin(\theta)\,\ddot{y} + \cos(\theta)\,\ddot{z} + \sin(\psi)\,\ddot{\theta} = -2\cos(\psi)\,\dot{\theta}\dot{\psi} - \varepsilon\nu\sin(\psi)\,\dot{\theta} \qquad (1d)$$

$$-\sin(\psi)\,\ddot{x} + \cos(\psi)\cos(\theta)\,\ddot{y} + \cos(\psi)\sin(\theta)\,\ddot{z} + \ddot{\psi} = \sin(\psi)\cos(\psi)\,\dot{\theta}^2 - \varepsilon\nu\dot{\psi} \qquad (1e)$$

where over-dot denotes derivative with respect to non-dimensional time τ. The parameter ε scales the magnitude of the nonlinear terms of the elastic forces acting on the external mass, as well as the coupling strength between the internal rotator and the external mass, ν is the viscous damping coefficient. The symmetric springs are considered to excite the fundamental $(1:1:1)$ resonant interaction between the three orthogonal directions of the external mass.

The numerical simulations corresponding to a system exhibiting complete, bi-directional, out-of-plane energy exchange between the $x - y$ plane oscillations of the external mass and the orthogonal $z -$ direction is shown in Fig 2 corresponding to $\nu = 0$. The high frequency oscillations along these directions are observed to be slowly modulated. Whenever the amplitude of oscillation along x and y directions reduces to zero, the amplitude of oscillation along the z direction is maximum and vice versa. The observed energy exchange is recurrent and bi-directional.

The second set of numerical simulations corresponds to a system exhibiting complete uni-directional, out-of-plane energy exchange corresponding to $\nu \neq 0$. In Fig. 3 we present the response corresponding to initial energy localization in the $y-z$ plane with uni-directional energy exchange to the $x-$ direction. At steady state, the oscillations of the external mass in the $y-z$ plane is completely suppressed and the energy is entirely entrapped in oscillations along $x-$ direction. It can be observed that the response of the slow-flow model matches extremely well with the original system (1) in both these cases.

Conclusions

We have considered low-energy regimes of locally resonant, 3D nonlinear oscillator comprising of an external mass coupled to an internal rotator. Using the regular multi time scale analysis, we derived the slow-flow model. Analysis of the slow-flow model revealed the regimes of uni- and bi-directional, partial and complete energy channeling exhibited by the system. The undamped slow-flow model is found to be fully integrable which enables significant reduction in complexity for certain non-stationary regimes of complete bi-directional energy flow. These symmetries are invoked to derive global analytical description of the formation and bifurcation of highly non-stationary regimes of complete bi-directional energy channeling between the in-plane and the out-of-plane oscillations of the external mass.

The second part of the study considers uni-directional energy channeling corresponding to damped system. Analysis of the slow-flow model reveals the regimes of transient irreversible energy channeling leading to complete 3D reorientation of the oscillations of the external mass. The uni-directional mechanism is manifested by permanent energy locking in steady-state oscillations orthogonally oriented to the plane of initial excitation. The discussed methodology can be extended to the analysis of wave propagation and redirection in 1D and 2D lattice of unit-cells and has applications in predictive design of seismic metamaterials and material systems for wave channeling.

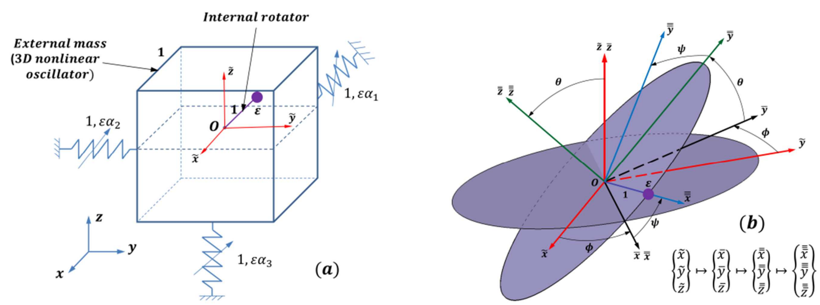

Figure 1: a) Schematic of the unit-cell model with internal rotator b) Euler angles to define the coordinates of the internal rotator

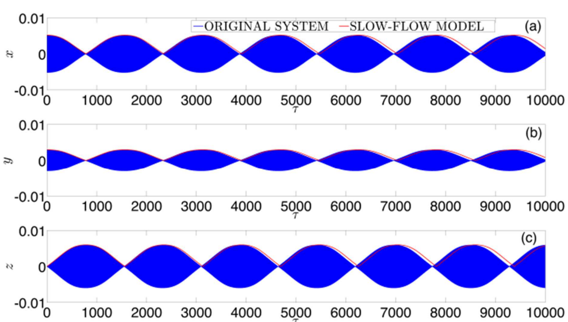

Figure 2: Response corresponding to bi-directional complete energy channeling for $x(0) = 0.005196, y(0) = 0.003, z(0) = 0, \theta(0) = 1.3717,\ \psi(0) = 1.2408$. a) $x-$ b) $y-$ c) $z-$ displacement of external mass.

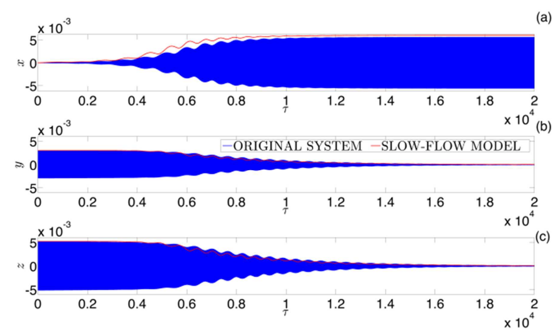

Figure 3: Response corresponding to uni-directional complete energy channeling for $x(0) = 0, y(0) = 0.003, z(0) = 0.0052, \theta(0) = \pi/3, \psi(0) = \pi/256,\ \nu = 0.15$. a) $x-$ b) $y-$ c) $z-$ displacement of external mass.

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