Nonlinear Reduced-Order Modeling for Controlled Aeroelastic Systems

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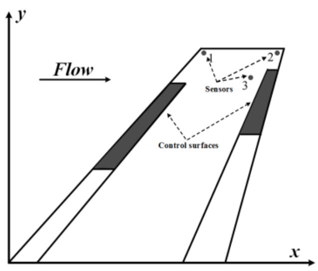
Summary. This paper presents a holistic nonlinear model order reduction methodology for controlled aeroelastic models of aerospace structures. The most attractive achievement of the methodology is proposing a low-dimensional mathematical model to represent the coupling dynamic characteristics of the structural dynamics of flexible aircraft and nonlinear aerodynamics of computational fluid dynamics-based aerodynamic systems. To demonstrate the accuracy of the reduce-ordered mathematical model in predicting aeroelastic behaviors, the Benchmark Active Control Technology wing is selected as the testbed. The comparison between the present reduce-ordered mathematical model and computational fluid dynamics-based model shows that the present nonlinear reduce-ordered aeroelastic model can accurately represent the aeroelastic behaviors of the aerospace structures.

Introduction

Aeroelastic performance of aerospace systems is characterized by the interaction between aerodynamics, structural dynamics, and active control dynamics. The design of modern aircraft utilizes state-of-the-art materials and lightweight structures to achieve better maneuverability and lower fuel consumption. For such aerospace structures, however, the flutter instability, nonlinear aeroelastic and aeroservoelastic (ASE) problems must to be investigated. In addition, in transonic flight regime, the control systems interactions with aerodynamic nonlinearity can result in more complicate ASE problems. To address the above challenges, high-fidelity ASE model coupling the nonlinear aerodynamics with structural models needs to be developed. Computational fluid dynamics (CFD)-based ASE analysis enables a direct insight into the aforementioned problems, its prohibitive computational cost, as well as difficulty to deploy controllers with high-state-order models render it impractical for integration in the design environment involving concurrent ASE analysis and control synthesis and design. Hence, it is desirable to establish an efficient and accurate representation of the transonic ASE systems to capture the physics of the transonic aerodynamic responses. The objective of this study is to develop a reduced-order modeling approach for nonlinear aeroelastic systems.

Structural Dynamic Modeling for Aircraft Structures with Control Surfaces

In this section, the mathematical description for aircraft structures with control surfaces is presented. The control surfaces, such as aircraft ailerons and missile rudders, are used as control inputs to suppress the aeroelastic vibrations. The present paper focuses on ASE modeling for aeroelastic control and the six degree-of-freedom rigid modes are ignored.



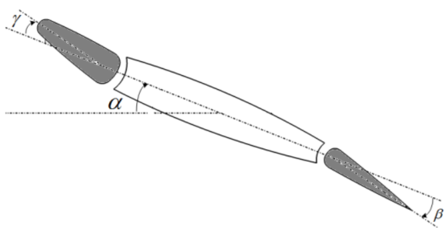


Fig. 1 Sketch for Structural Dynamic Modeling

The physical displacement of the wing model at an arbitrary position (x, y) can be expressed by

$$u(x,\ y,\ t)=\sum_{i=1}^{n}\varphi_{q_i}(x,\ y)q_i(t)+\varphi_\gamma(x,\ y)\gamma(t)+\varphi_\beta(x,\ y)\beta(t) \tag{1}$$

where φ_{q_i}, φ_γ, and φ_β are the mode shapes of the wing model, leading-edge control surface, and trailing-edge control surface, respectively. Thus, the kinetic energy T and potential energy U of the wing model can be written as

$$T=\frac{1}{2}\iint_S \mu(x,\ y)u^2(x,\ y,\ t)dxdy,\ U=\frac{1}{2}\sum_{i=1}^{n}\left[\left(\iint_S \mu(x,\ y)\varphi_{q_i}^2(x,\ y)dxdy\right)\omega_{q_i}^2 q_i^2\right]+\frac{1}{2}K_\gamma\left(\gamma_c-\gamma\right)^2+\frac{1}{2}K_\beta\left(\beta_c-\beta\right)^2 \tag{2}$$

where $\mu(x,y)$ and ω_{q_i} are the mass distribution and ith natural frequency of the wing model, respectively. The stiffness terms K_γ and K_β are the spring constants representing the flexibility in the structure supporting the actuators. The strain energy associated with the leading-edge and trailing-edge control surfaces are based on the difference between the control commands and the actual deflections. The virtual work of the unsteady dynamic forces can be expressed by

$$\delta W_a = q_\infty \iint_S C_P\left(x, y\right) \delta u\left(x, y, t\right) dxdy \tag{3}$$

By combing the Eqs. (1)-(3), the aeroelastic equations of motion can be easily expressed by

$$\begin{bmatrix} \boldsymbol{M}_{qq} & \boldsymbol{M}_{q\gamma} & \boldsymbol{M}_{q\beta} \\ \boldsymbol{M}_{q\gamma}^{\mathrm{T}} & M_{\gamma\gamma} & 0 \\ \boldsymbol{M}_{q\beta}^{\mathrm{T}} & 0 & M_{\beta\beta} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}}(t) \\ \ddot{\gamma}(t) \\ \ddot{\beta}(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_{qq} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & K_\gamma & 0 \\ \boldsymbol{0} & 0 & K_\beta \end{bmatrix} \begin{bmatrix} \boldsymbol{q}(t) \\ \gamma(t) \\ \beta(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{0}_{n\times 1} & \boldsymbol{0}_{n\times 1} \\ K_\gamma & 0 \\ 0 & K_\beta \end{bmatrix} \begin{bmatrix} \gamma_c(t) \\ \beta_c(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{Q}_{qq}(t) \\ Q_{\gamma\gamma}(t) \\ Q_{\beta\beta}(t) \end{bmatrix} \tag{4}$$

where $\boldsymbol{Q}_{qq}(t)$, $Q_{\gamma\gamma}(t)$ and $Q_{\beta\beta}(t)$ are the nonlinear aerodynamic terms which cannot be obtained analytically in a transonic flight regime.

Reduced-Order Modeling for Transonic Aerodynamic Systems

The aerodynamic computations for transonic aeroelastic modeling are significantly more complicated because of the nonlinear behaviors introduced by the deflection of control surface, shock vibration and boundary-layer separation. The CFD-based aerodynamic computations can effectively capture the nonlinear behaviors, but the huge time consumptions prevent its applications in design of aeroelastic controller and closed-loop aeroelastic analysis. The concept of reduced-order modeling for CFD-based aerodynamic systems suggests that the input-output relation of a complex CFD system can be represented by a relatively simple mathematical model, which is the reduced-order model (ROM). Feeding an arbitrary input signal through the ROM is far more efficient than feeding it through the full CFD analysis, and yet the response captures the physical characteristics of the nonlinear system [1, 2]. In this study, the system identification-based ROM approach is used to construct aerodynamic ROM for transonic aeroelastic modeling. Figure 2 gives the block diagram of the nonlinear ROM approach. As shown in the figure, the nonlinear ROM consists of a sum of Wiener-type cascade models, which can be represented by a linear state-space model followed by a nonlinear neural network. Based on the input and output samples obtained via CFD computations, the parameters of the Wiener-type model can be estimated via nonlinear system identification method [3].

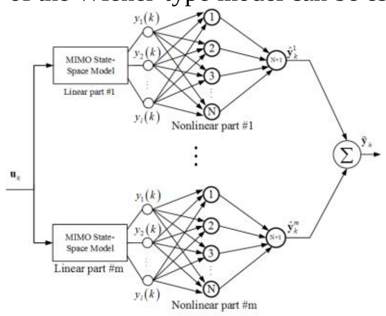


Fig. 2 Reduced-order model for aerodynamic systems

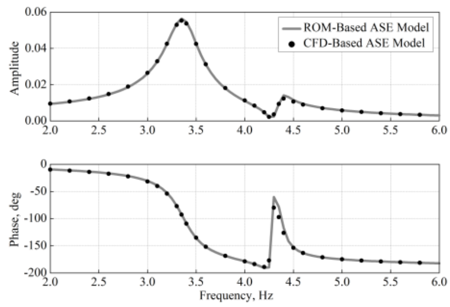


Fig. 3 Comparison of the frequency responses

Preliminary Numerical Results

For the validation of transonic ASE modeling, this study focuses on the representation of the frequency responses between the deflection of control surface and modal displacements. Figure 3 illustrates the comparisons of frequency responses obtained via ROM-based and CFD-based methods, respectively. The operating condition corresponds to a Mach number of 0.8 and a pre-flutter dynamic pressure of 125 psf. As shown in the figure, the ASE frequency responses of the wing model predicted via the ROM-based aeroelastic modeling method are in good agreement with the CFD method.

Conclusions

This paper proposed a novel reduced-order modeling approach for nonlinear aeroleastic systems. The most attactive feature is that it provides an efficient and accurate mathimatical modeling method for nonlinear aerodynamic forces which was computed via CFD-based computation in the previous studies. Based on the present reduced-order aeroelastic models, the nonlinear aeroelastic analysis and controller design can be easily implemented. The preliminary numerical results demonstrate that the aeroelastic behaviors of the aircraft structures can be effectively predicted.

References

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