Analysis of Synchronization in Mutually Coupled MEMS Oscillators Via Surface Acoustic Waves Using a Simplified Non-linear Model

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Summary. When a MEMS oscillator vibrates it loses energy to the substrate in the form of surface acoustic waves(SAW). This SAW which is of delay type could be used to couple and synchronize the oscillators. SAW coupled MEMS oscillators which are parametrically excited exhibiting limit cycle oscillations and hysteresis can be modeled in a simplified form as Mathieu-Van der-pol duffing oscillators with delay coupling. This work concerns itself with the synchronization analysis of two linear reactive delay coupled Mathieu-Van der Pol-Duffing oscillators. The synchronization phenomenon in this system is analyzed in detail using analytical and numerical means. The Krylov-Bogoliubov averaging method has been used to arrive at the phase equation for the system. The solutions of the phase equation for different parameter values and initial conditions are validated by comparing them with numerical results obtained by solving the original model. The parameter values corresponding to the synchronization regimes are arrived at from the phase equation and are validated by numerical studies. The stability and bifurcation analysis of the derived phase equation carried out shows that there exists a saddle node bifurcation via which a 1:1 sychronization orginates. A transcritical bifurcation which brings about a flip from in-phase synchronization to anti-phase synchronization is also observed.

Introduction

Micro electro mechanical system (MEMS) based oscillators have been an active area of research in the last few decades. Due to their minuscule size and lower power consumption they find a wide range of applications from accelerometers, gyroscopes, micro mirrors, gas sensors etc. to lab on chip [1].
Certain non-linear phenomena like jumps, hardening and softening, bifurcations, limit cycle oscillations etc. are commonly seen in MEMS structures [2,3]. One can obtain frequency or phase locking by coupling two oscillators which undergo limit cycle oscillations. This nonlinear phenomenon is known as synchronization/entrainment [4]. As the MEMS device vibrates it loses energy to the substrate via anchor, causing disturbance in the substrate which propagates away from the oscillator in the form of stress waves [3]. These stress waves consist of bulk acoustic waves (BAW) and surface acoustic waves (SAW). A major portion (67%) of the energy is in the SAW [5]. This SAW which otherwise would go waste can be used to couple the oscillators and synchronize them. This coupling due to SAW will be of delay type. This work is concerned with analysis of synchronization in acoustically coupled micro-oscillators exhibiting phenomena such as limit cycle and hysteresis. These kinds of oscillators are generally forced parametrically by modulating incident laser beam. As a simplified version a Mathieu-Van der pol-Duffing equation with delay coupling can be used to model such systems.
This paper studies synchronization of two mutually delay coupled Mathieu-Van der Pol-Duffing oscillators wherein coupling is a linear reactive one. Synchronized MEMS oscillators have potential applications in signal processing, neural computing, sensing and other fields.

Numerical Analysis

The governing equation for two linear reactive delay coupled Mathieu-Van der pol-Duffing oscillators is given by

$$\ddot{x}_1 - \lambda(1-x_1^2)\dot{x}_1 + (\omega_1^2 + cos(\omega t))x_1 + \beta x_1^3 + B_R(x_1(t-\tau) - x_2(t-\tau)) = 0 \quad (1)$$

$$\ddot{x}_2 - \lambda(1-x_2^2)\dot{x}_1 + (\omega_2^2 + cos(\omega t))x_2 + \beta x_2^3 + B_R(x_2(t-\tau) - x_1(t-\tau)) = 0 \quad (2)$$

Where, B_R is the coupling strength, ω_1 and ω_2 are eigen frequencies of the system, τ is the time delay, β is the cubic nonlinearity, λ is the nonlinear damping and ω is the parametric forcing frequency. The Eq(1) and Eq(2) are delayed differential equations and are solved numerically using DDE23 algorithm in MATLAB to obtain its response. Lissajous figure is used to check whether system is synchronized or not.
The following observations were made from numerical analysis:- For larger detuning if coupling strength is low, the system will not synchronize while for same detuning, the system can be synchronized by increasing the coupling strength. If a symmetric initial condition is given to the system i.e. both the oscillators are given initial displacements in the same direction, the system is in in-phase synchronization. One may expect anti-phase synchronization when an anti-symmetric initial condition is applied to the system i.e. when one oscillator is given exactly the opposite initial condition with respect to the other. But the analysis showed that the system remains in in-phase synchronization itself in the case. Another fact observed is that the synchronization remains stable to local perturbations in the initial conditions.

Analytical Analysis

The aim here is to derive a single equation which can capture qualitatively the dynamics of synchronization. The following assumptions are made for deriving the analytical solution:- the coupling strength B_R is not large when compared to the

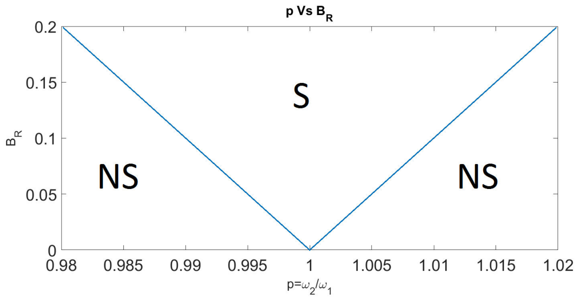

Figure 1: 1:1 synchronization tongue for the system, where p=$\frac{\omega_2}{\omega_1}$, S:- Synchronized region, NS:- Non Synchronized region

amplitude of unperturbed oscillations, the parametric excitation frequency ω is only slightly different from ω_1 and ω_2 and the frequency detuning Δ is small.

Method of averaging has been used to derive the analytic approximation. The slow flow equation for the phase thus obtained is given by

$$\dot{\theta} = \Delta + \frac{3\beta}{8\omega}(A_2^2 - A_1^2) + \frac{4B_R}{8\omega}(\frac{A_2}{A_1}cos(\omega\tau - \theta) - \frac{A_1}{A_2}cos(\omega\tau + \theta)) \tag{3}$$

Where, θ is the phase difference between the oscillators and τ is the delay. By assuming a symmetric solution i.e. $A_1 = A_2 = A$, Eq(3) becomes independent of amplitude terms and gets simplified to

$$\dot{\theta} = \Delta + \frac{B_R}{\omega}sin(\omega\tau)sin(\theta) \tag{4}$$

The Eq(4) is the **phase dynamics equation** for the system and the synchronization dynamics of the system can be studied by solving this single equation. For synchronization to take place $\dot{\theta} = 0$, i.e. θ should become constant after some initial transience. Thus one could check whether the system is synchronized or not by plotting θ Vs $time$ through numerical integration of Eq(4).

Stability and Bifurcation Analysis of the Phase Equation

The fixed points of the obtained phase dynamics equation are $\theta_1 = \arcsin(-\frac{\Delta\omega}{B_R sin(\omega\tau)})$ and $\theta_2 = \pi - \arcsin(-\frac{\Delta\omega}{B_R sin(\omega\tau)})$. The Fixed points θ_1 and θ_2 exist as long as $\Delta\omega \leq B_R sin(\omega\tau)$. By performing linear stability analysis it is found that θ_1 is an unstable fixed point and θ_2 is a stable fixed point, assuming $sin(\omega\tau)$ to be positive. When the equality$\Delta\omega = B_R sin(\omega\tau)$ is satisfied these fixed points θ_1 and θ_2 will merge together and get annihilated, giving rise to a **Saddle node bifurcation**. This means that there is no longer a constant phase difference between the oscillators. So in the (B_R, Δ) plane the curve $\Delta = \pm\frac{B_R sin(\omega\tau)}{\omega}$ gives the boundary for 1:1 synchronization fig(1). It is observed that after a certain value of τ the fixed points θ_1 and θ_2 interchange their stability giving rise to a **Transcritical bifurcation**. This accounts for the flip from in-phase synchronization to out-of-phase synchronization when τ varied.

Conclusion

The synchronization of two linear reactive delay coupled Mathieu-Van der Pol-Duffing oscillators is studied as a simplified model for SAW coupled MEMS oscillators. The equation for phase dynamics of the model was derived and it showed matching results with the numerical one. An expression connecting B_R,Δ,ω and τ,giving the borderline for 1:1 synchronization is obtained. The stability analysis of the fixed points for the phase equation is carried out and it is found that one fixed point is a stable while the other is unstable. These two fixed points merge together and they disappear via saddle node bifurcation. It is observed that there is a flip from in-phase synchronization to anti-phase synchronization via transcritical bifurcation by varying the delay of the system.

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