Transient Responses And Bifurcation Behavior of A Piecewise Smooth Rotor/Stator Rubbing System under Noise Excitation

Jun Jiang*, Ling Hong* and Zigang Li**
* *State Key Lab for Strength and Vibration, Xi'an Jiaotong University, Xi'an 710049, China*
** *School of Science, Xian University of Science and Technology ,Xi'an 710024, China*

Summary. In this paper, an effective approach based on the parallel Generalized Cell Mapping method with evolving probabilistic vector, or GCM with EPV in short, is first introduced, which can efficiently capture the large probability transition in high-dimensional and non-smooth systems. Then, the noise-induced transition behaviors in a piecewise smooth rotor/stator rubbing system subject to additive noise are investigated, with the knowledge on its deterministic global characteristics. It is found that noise-induced explosive and dangerous bifurcations are induced when the noise intensity exceeds some critical values within the multi-stable regions of the rubbing rotor system. The noise may either produce beneficial effect to decouple the rubbing rotor or induce a several times increase in the rotor deflection depends on system parameters and noise intensity. The results may be helpful to understand the effect of the noise on the qualitative change in the responses of the rotor/stator rubbing system.

Generalized Cell Mapping Method with Evolving Probabilistic Vector

Based on the Generalized Cell Mapping method [1], the probability evolution of the stochastic system with temporally homogeneous Markov process in a continuous state spaces $\mathbf{R}^N$ can be described by a homogeneous Markov chain in the cell space

$$\mathbf{p}(n+1) = \mathbf{P}\cdot\mathbf{p}(n) \tag{1}$$

where $\mathbf{p}(n)$ denotes the probabilistic vector describing the probability of each cell at nth step, and $\mathbf{P}$ the one-step transition probability matrix of the stochastic system. The element P_{ij} and $p_i(n)$ can be determined by following formulae

$$P_{ij} = \int_{C_i} p(\mathbf{x},t|\mathbf{x}_j,t_0)d\mathbf{x} = \int_{C_i} p(\mathbf{x},\tau|\mathbf{x}_j,0)d\mathbf{x}, \qquad p_i(n) = \int_{C_i} p(\mathbf{x},n\tau)d\mathbf{x} \tag{2}$$

where τ=t-t_0 denotes a mapping time step; C_i is the domain occupied by ith cell in $\mathbf{R}^N$, and $p(\mathbf{x}, \tau|\mathbf{x}_j, 0)$ and $p(\mathbf{x}, n\tau)$ represent the one-step transition probability and the probability under n-steps mapping in $\mathbf{R}^N$, respectively.

Borrowing the idea from Point Mapping under Cell Reference method [2], the cells in the chosen region will be classified into *active cells* and *inactive cells*. An active cell represents the cell whose probability density function (PDF) is within the prescribed fiducial probability, and an inactive cell is the cell whose PDF is outside the prescribed fiducial probability. In simulation, inactive cells will not be included into the probabilistic vector in the computation. So the probabilistic vector $\mathbf{p}(n)$ in the present method is no longer a vector with a fixed length N_c as in the traditional GCM, the length of the probabilistic vector $\mathbf{p}(n)$ will vary and equal the number of active cells N_a at nth-step mapping from a given initial probability distribution. Let r denotes the inactive cell whose probability is outside the fuducial probability, the evolving probabilistic vector is then governed by

$$\begin{cases} P_{ij}\ p_j(n) = 0 & when \quad j = r \\ P_{ij}\ p_j(n) = p_i(n+1) & when \quad j \neq r \end{cases} \qquad i,j = 1,2,3,...N_c \tag{3}$$

In the simulation, the fiducial probability is prescribed as a threshold to distinguish the active cells and the inactive cells. The active cells can be determined by the sum of the ordered probability.

$$P_f = \sum_i \sum_{j\neq r} P_{ij} p_j(n) \tag{4}$$

Noise-induced Bifurcations in Piecewise Smooth Rubbing Rotors

The non-dimensional governing equations of the rotor/stator rubbing system are written in the form:

$$\begin{cases} \ddot{X} + 2\zeta\dot{X} + \beta X + \Theta(1-\dfrac{R_0}{R})(X - \mu\text{sign}(V_{rel})Y) = \Omega^2\cos(\Omega t) + \varepsilon w_1(t) \\ \ddot{Y} + 2\zeta\dot{Y} + \beta Y + \Theta(1-\dfrac{R_0}{R})(\mu\text{sign}(V_{rel})X + Y) = \Omega^2\sin(\Omega t) + \varepsilon w_2(t) \\ V_{rel} = R_{disk}\Omega + R\omega_b \end{cases} \tag{5}$$

where X, Y are the dimensionless deflections of the rotor, ζ and β denote respectively the damping ratio and the stiffness ratio, μ is the friction coefficient, $V_{\rm rel}$ represents the relative velocity between the rotor and the stator at the contact point, ω_b=ω_w/ω_s is the dimensionless frequency ratio of the whirling frequency and natural frequency, R_{disk} the disk radius, e the mass eccentricity, Ω the rotating speed of the rotor. The Heaveside function Θ=0 when $R<R_0$, and Θ=1 when $R\geq R_0$ with $R=\sqrt{X^2+Y^2}$ and R_0=r_0/e is the initial clearance between the rotor and the stator.

In Eqn. (5), ε presents the noise intensities and $w_i(t)$ (i=1,2) is Gaussian white noise with its mean and correlation function satisfying

$$E[w_i(t)] = 0, \quad E[w_i(t)w_j(t+t')] = \begin{cases} \delta(t'), & i = j \\ 0, & i \neq j \end{cases} \tag{6}$$

The chosen region in the four-dimensional state space is **D**={-25< X <25, -25< X' <25, -25< Y <25, -25< Y' <25}, which will be covered by 200×100×200×100 cells. The total number of cells is 400000000, which is a considerable number of cells for the traditional GCM and will be very time-consuming. By using GCM with EPV, the fiducial probability is chosen to be P_f = 0.95 in the computations. The noise-induced phenomena in the multi-stable parameter regions of the rotor/stator rubbing system are studied. Both the noised-induced explosive and dangerous bifurcations are observed. An example of *noise-induced explosive bifurcation* is shown below since the stochastic response increases the size of the response region abruptly when the noise intensity exceeds a certain critical value.

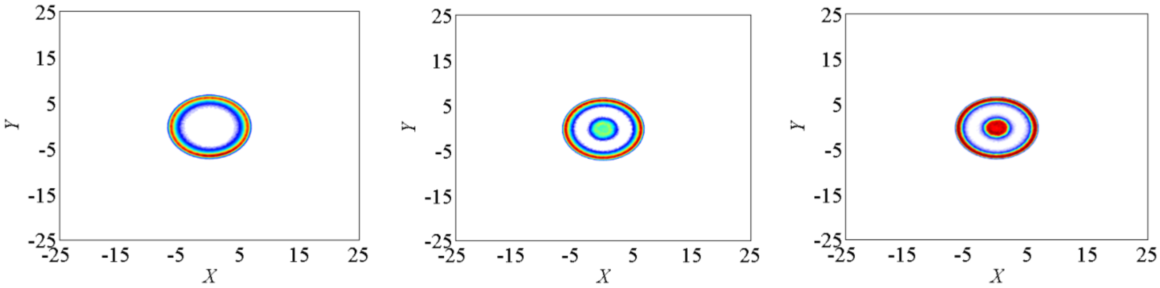

Fig. 1 Pseudo-color images of the evolving PDFs of rubbing system initially from backward whirl motion: (a) t =10T; (b) t =100T; (c) t =5000T, when ε=0.25, Ω=0.30, ζ=0.05, β=0.04, μ=0.08 and R_{disk}=20.

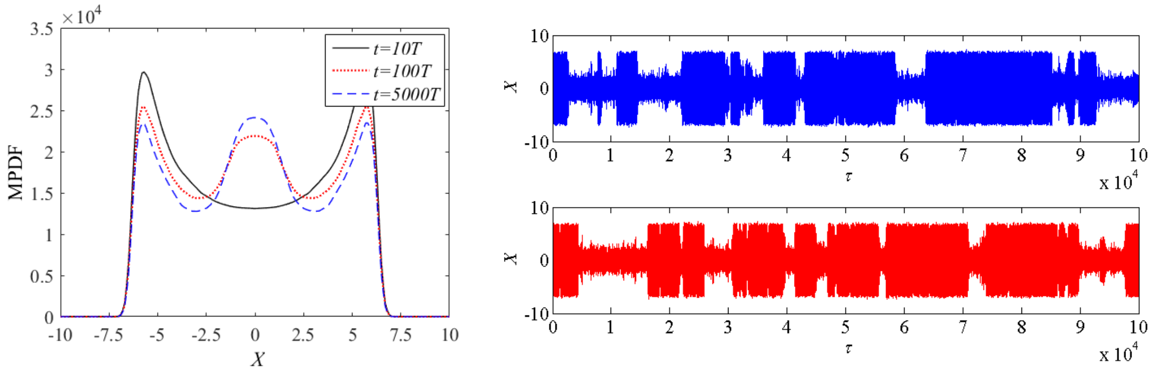

Fig. 2 Evolving MPDF in x-direction (left) and time history of response samples (right) initially starting from around backward whirl motion with a larger amplitude, when ε=0.25, Ω=0.30, ζ=0.05, β=0.04, μ=0.08 and R_{disk}=20.

Concluding Remarks

By using the proposed parallel Generalized Cell Mapping method *with evolving probabilistic vector*, the transient responses and the bifurcations of a four dimensional piecewise smooth rotor/stator rubbing system under noise excitation are investigated efficiently. It is found that that the rotor/stator rubbing system might become very sensitive to the noise disturbance when the system is in the multi-stable regions. In some parameter range, noise may benefit the rotor system by decoupling the rotor from the stator. Noise may induce unexpected large responses of the rotor system. It is thus valuable to well understand the noise-induced behaviors in the piecewise smooth rotor/stator rubbing system in order to take use of its beneficial effect but avoid the undesired destructive responses.

Acknowledgments

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References

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