Controlling Multistability in Vibro-Impact Systems

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Summary. This work concerns the control of multistability in vibro-impact systems. Special attention is given to two control issues in the framework of multistable engineering systems, namely, the switching between coexisting attractors without altering the system's main parameters, and the avoidance of grazing-induced chaotic responses. A periodically excited oscillator with soft impacts and a vibro-impact capsule system are used as case studies, where the main concern is to investigate numerically the effectiveness of the proposed control methods. In addition, a detailed study of the robustness of the control schemes is carried out via continuation methods for non-smooth dynamical systems, which are also used to identify a region in the parameter space where the control methods can be applied.

Introduction

Multistability is an inherent property referring to the systems that exhibit coexistence of several stable solutions for a given set of parameters, and the development of robust control methods based on this feature is a topic of ongoing investigation, see e.g. [1, 2, 3]. Our main concern in this work is to address two control issues, namely, the advantageous use of multistability in order to optimize the operation of vibro-impact systems and the avoidance of chaotic responses in the vicinity of grazing events. For these purposes, we implement a control scheme based on a feedback (closed-loop) controller, and investigate in detail its effectiveness for two case studies, namely, a periodically excited soft impact oscillator [4] and a vibro-impact capsule system [5].

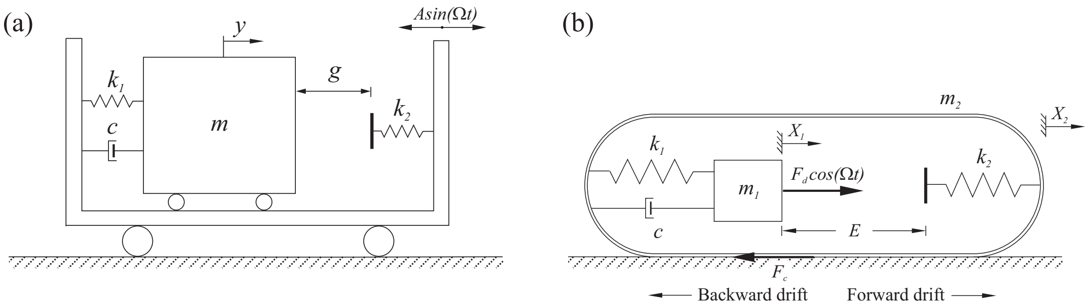

Figure 1: Physical models of (a) the impact oscillator and (b) the vibro-impact capsule system.

Periodically excited oscillator with soft impacts

In the first example, we will study the multistability of a periodically excited oscillator with soft impacts as shown in Fig. 1(a). Here, we employ a position-velocity feedback control law which is given as $u(\tau) = \epsilon\left[(x_d(\tau) - x(\tau)) + \lambda(v_d(\tau) - v(\tau))\right]$, where ϵ stands for the coupling strength between the impact oscillator and the controller, λ is a constant weighting factor, and $\big(x_d(\tau), v_d(\tau)\big)$, $\tau \geq 0$, gives the position and velocity of the target solution to which the system should be driven. Fig. 2(a) shows the basins of attraction for which a period-2, period-3, period-5, and period-8 coexist. Our numerical study reveals that the basins of attraction are fractally interwoven due to grazing events. Here, our aim is to drive the oscillator from any coexisting attractor to the period-2 attractor denoted by blue dots with green basin. Fig. 2(b) presents the control result from the period-8 to period-2 attractor, and Fig. 2(c) shows the control result from the period-5 to period-2 attractor. In the considered cases, the controller was activated at $\tau \approx 195$, and the oscillator settled down quickly to the desired period-2 attractor thereafter. The pictures also show that a the time the control is activated an impulsive perturbation is produced by the driving signal, whose amplitude is significantly larger than the harmonic excitation.

Vibro-impact capsule system

The second example considers a two-degree-of-freedom capsule system depicted in Fig. 1(b). We consider a multistable scenario of the system as shown in Fig. 3(a), where four attractors coexist. In order to control its forward and backward motion, we employ the position feedback control law $u = k_p|\,x_2 - x_1|$, where k_p is a proportional control gain. The result with the control law can be observed in Fig. 3(b). The main difference is that the unwanted solutions disappeared, and only the desired forward and backward drifting solutions persist,

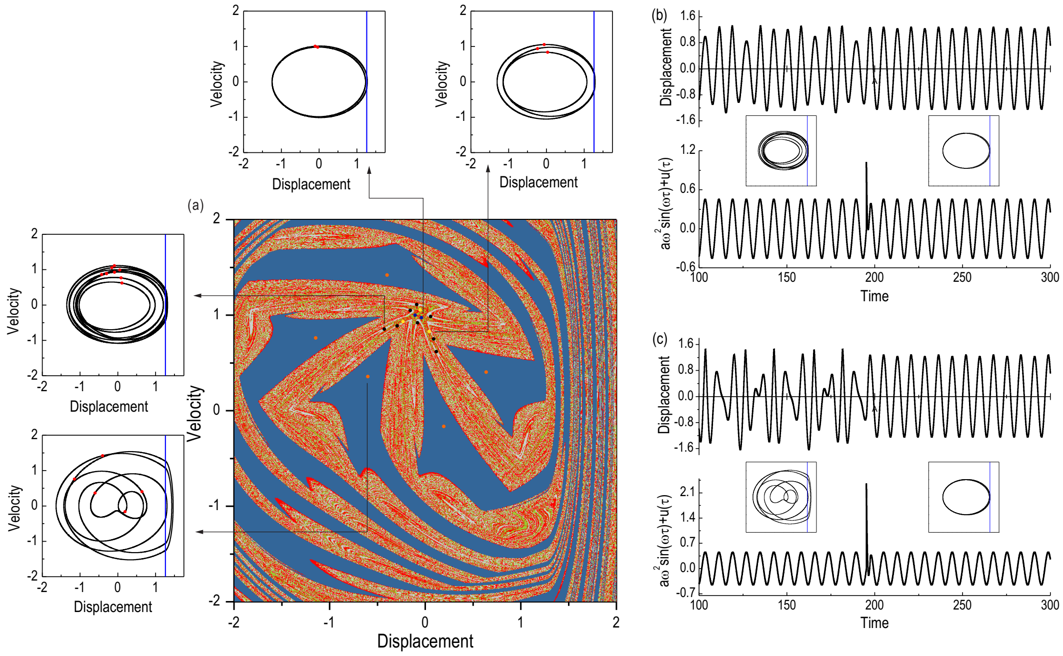

Figure 2: (a) Basins of attraction for the impact oscillator with $\zeta = 0.01$, $e = 1.26$, $a = 0.7$, $\omega = 0.8044$, and $\beta = 29$. There are four stable attractors, the period-2 (blue dots with green basin), the period-3 (yellow dots with red basin), the period-5 (orange dots with blue basin), and the period-8 attractors (black dots with grey basin) coexist. By using the control law, the system is controlled from (b) the period-8, and (c) the period-5 attractor to the period-2 attractor.

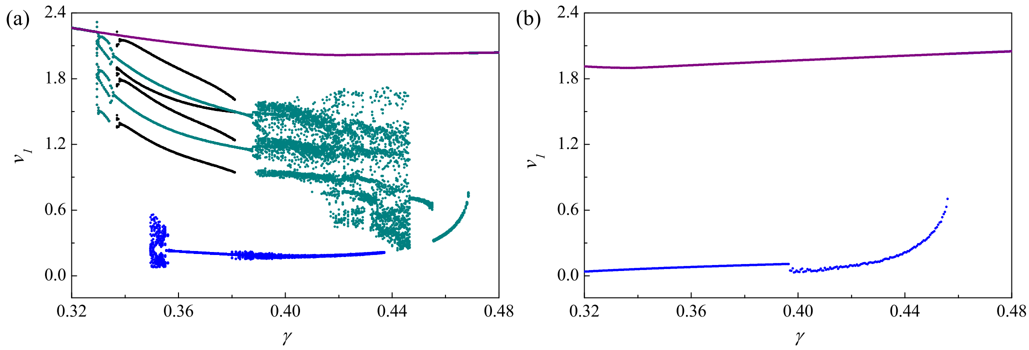

Figure 3: Bifurcation diagrams for the capsule system constructed for varying mass ratio γ, (a) without control ($k_p = 0$) and (b) with control ($k_p = -0.2$), calculated for $\beta = 3$, $\xi = 0.05$, $\delta = 0.02$, $\alpha = 1.6$, $\omega = 1.1$, and $v_s = 0.1$.

meaning that the system switched from the multistable to a bistable dynamical scenario. Another effect is that the window of existence of the solution with backward drift (blue line) has increased to $\gamma \in [0.32, 0.456)$.

Conclusions

From a practical point of view, multistability introduces a twofold effect in engineering systems. On one hand, this phenomenon has to be avoided to prevent costly failures by stabilizing the desired state against a noisy environment when designing a commercial device with specific characteristics. On the other hand, coexistence of several stable solutions offers great flexibility in the performance of engineering systems, because the system behavior can be changed without altering its major control parameters. We have studied both control issues using a periodically excited oscillator with soft impacts and a vibro-impact capsule system. In order to gain an insight into the robustness of the proposed control methods, numerical continuation methods were also applied to identify a region in the parameter space in which the proposed controllers can be applied.

References

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