Distributed adaptive synchronization control for networked Lagrange system with dynamic friction compensation

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Summary.We study the problem of synchronization of networked Lagrange system with dynamic friction compensation. To obtain the accurate description of physical friction, Lugre friction model is introduced to the controlled system. The tracking control algorithm for uncertain parameters system are provided, which has the capability of adapting changes due to external disturbance and aging of material by learning from the tracking error. In addition, the proposed algorithm has a weak assumption on the value of control gain and is easy to be realized. Simulations are provided to show the effectiveness of the proposed tracking algorithm.

Introduction

In recent decades, synchronization control has attracted much attention from researchers. Various control strategies are proposed based on the Euler Lagrange approach and LaSalle's invariance principle [1]. It is noted that the friction is not considered or simply expressed by means of a static model in most references. According to [2] and [3], static friction model neglects the deformation of the contact surface and is unable to explain the hysteretic behaviour, which may lead to tracking error in high precision control. Lugre model [2] is selected in this work because it does not only incorporate features of classical static friction model, but also has the capability to explain most friction behaviours. We propose the tracking algorithm by introducing dual-observer [4] to networked Lagrange system with uncertain parameters. By applying the proposed tracking algorithm, synchronization can be rapidly achieved in simulations.

System equation and main results

Suppose that the networked Lagrange mechanical system has one leader, i.e. agent 0 as an extra agent and N followers, named as agent 1 to agent N. The topology of the information flow among followers can be represented by a graph. Consider the directed graph $\mathbb{G}=(\mathbb{V},\mathbb{E})$, where $\mathbb{V}=\{1,2,\cdots,N\}$ is the set of nodes and $\mathbb{E}\subseteq\mathbb{V}\times\mathbb{V}$ is the set of edges. The edge $(i,j)\in\mathbb{E}$, where $i,j\in\mathbb{V}$, indicates that agent j can obtain information from agent i. Similarly, define $\overline{\mathbb{G}}=(\overline{\mathbb{V}},\overline{\mathbb{E}})$, where $\overline{\mathbb{V}}=\{0,1,2,\cdots,N\}$, being the set consisting the leader and all followers. $\mathbf{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$ and $\mathbf{B}=diag(b_{10},b_{20},\cdots,b_{N0})$ where

$$a_{ij}=\begin{cases}>0 & ,(j,i)\in\mathbb{E}\ \ and\ \ i\neq j\\ 0 & ,(j,i)\notin\mathbb{E}\ \ or\ \ i=j\end{cases},\quad b_{i0}=\begin{cases}>0 & ,(0,i)\in\overline{\mathbb{E}}\\ 0 & ,(0,i)\notin\overline{\mathbb{E}}\end{cases}. \tag{1}$$

By using Euler-Lagrange formalism, the dynamic model of N agents n-degree networked Lagrange system can be described as follows:

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i+\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i+\mathbf{G}_i(\mathbf{q}_i)+\mathbf{T}_{Li}=\boldsymbol{\tau}_i\quad,\quad i=1,2,\cdots,N, \tag{2}$$

where $\mathbf{q}_i=\begin{bmatrix}q_{i1} & q_{i2} & \cdots & q_{in}\end{bmatrix}^T\in\mathbb{R}^n$ is the vector of generalized coordinates of agent i $(i=0,1,\cdots,N)$, $\dot{\mathbf{q}}_i$ and $\ddot{\mathbf{q}}_i$ are the generalized velocity and generalized acceleration, respectively. $\mathbf{M}_i(\mathbf{q}_i)\in\mathbb{R}^{n\times n}$ is a positive definite inertia matrix. $\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\in\mathbb{R}^{n\times n}$ is the Coriolis matrix related with body rotation. $\mathbf{G}_i(\mathbf{q}_i)\in\mathbb{R}^n$ is the vector of gravitational force. $\mathbf{T}_{Li}=\begin{bmatrix}T_{Li1} & T_{Li2} & \cdots & T_{Lin}\end{bmatrix}^T\in\mathbb{R}^n$ represents the vector of Lugre friction. $\boldsymbol{\tau}_i\in\mathbb{R}^n$ is the vector of control input. Based on [5], the Lugre friction can be expressed by $T_{Lij}=\sigma_{0ij}z_{ij}+\sigma_{1ij}\dot{z}_{ij}+\sigma_{2ij}\dot{q}_{ij}$ where σ_{0ij}, σ_{1ij}, σ_{2ij} and z_{ij} represent stiffness coefficient, damping coefficient, viscous coefficient and internal state parameter updated by $\dot{z}_{ij}=\dot{q}_{ij}-z_{ij}|\dot{q}_{ij}|/g_{ij}(\dot{q}_{ij})$ where g_{ij} is a nonlinear function with respect to $\dot{q}_{ij}$. It should be noted that σ_{0ij} and σ_{1ij} are also called dynamic friction parameters due to their relation with the internal state parameter z_{ij}.

To design the tracking algorithm and guarantee the stability of the controlled system, we need the following assumptions: $\mathbb{G}$ has a spanning tree; $trace(\mathbf{B})>0$; all agents have access to their own generalized coordinates and velocity; the leader has no access to any other agent and the leader is able to deliver $\dot{q}_0$ and $\ddot{q}_0$ to all other agents; if $a_{ij}>0$, q_j is supposed to be delivered to agent i by agent j, and if $b_{i0}>0$, q_0 is supposed to be delivered to agent i by the leader; $g_{ij}(\dot{q}_{ij})$ is a known function. It can be proved that when the above assumptions hold, all followers will track the trajectory of the leader by applying the following adaptive tracking algorithm to system(2),

$$\boldsymbol{\tau}_i = \mathbf{Y}_i\left(\mathbf{q}_i, \dot{\mathbf{q}}_i, \boldsymbol{\eta}_i, \dot{\boldsymbol{\eta}}_i\right)\hat{\boldsymbol{\theta}}_i + \hat{\mathbf{T}}_{Li} - \mathbf{K}_{2i}\boldsymbol{\xi}_i \quad ,i = 1, 2, \cdots, N, \tag{3}$$

where $\mathbf{Y}_i$ is a known regression matrix with respect to system parameter, $\boldsymbol{\eta}_i = \dot{\mathbf{q}}_0 - \mathbf{K}_{1i}\left[\sum_{j=1}^{N} a_{ij}\left(\mathbf{q}_i - \mathbf{q}_j\right) + b_{i0}\left(\mathbf{q}_i - \mathbf{q}_0\right)\right]$, $\boldsymbol{\xi}_i = \dot{\mathbf{q}}_i - \boldsymbol{\eta}_i$ and $\hat{\mathbf{T}}_{Li}$ is updated as follows:

$$\hat{\mathbf{T}}_{Li} = diag\left(\hat{\mathbf{z}}_{0i}\right)\hat{\boldsymbol{\sigma}}_{0i} + diag\left(\dot{\mathbf{q}}_i - \boldsymbol{\Xi}_i\hat{\mathbf{z}}_{1i}\right)\hat{\boldsymbol{\sigma}}_{1i} + diag\left(\dot{\mathbf{q}}_i\right)\hat{\boldsymbol{\sigma}}_{2i} \quad ,i = 1, 2, \cdots, N.$$

Adaptive laws of other parameters are listed as follows:

$$\begin{array}{lll} \dot{\hat{\mathbf{z}}}_{0i} = \dot{\mathbf{q}}_i - \boldsymbol{\Xi}_i\hat{\mathbf{z}}_{0i} - \mathbf{K}_{3i}\boldsymbol{\xi}_i & \dot{\hat{\boldsymbol{\sigma}}}_{0i} = -\mathbf{K}_{5i}diag\left(\hat{\mathbf{z}}_{0i}\right)\boldsymbol{\xi}_i & \dot{\hat{\boldsymbol{\sigma}}}_{2i} = -\mathbf{K}_{7i}diag\left(\dot{\mathbf{q}}_i\right)\boldsymbol{\xi}_i \\ \dot{\hat{\mathbf{z}}}_{1i} = \dot{\mathbf{q}}_i - \boldsymbol{\Xi}_i\hat{\mathbf{z}}_{1i} + \mathbf{K}_{4i}\boldsymbol{\Xi}_i\boldsymbol{\xi}_i & \dot{\hat{\boldsymbol{\sigma}}}_{1i} = -\mathbf{K}_{6i}diag\left(\dot{\mathbf{q}}_i - \boldsymbol{\Xi}_i\hat{\mathbf{z}}_{1i}\right)\boldsymbol{\xi}_i & \dot{\hat{\boldsymbol{\theta}}}_i = -\mathbf{K}_{8i}\mathbf{Y}_i^T\boldsymbol{\xi}_i \end{array} \quad ,i = 1, 2, \cdots, N.$$

where $\boldsymbol{\Xi}_i = diag\left(\left|\dot{q}_{i1}\right| / g_{i1}\left(\dot{q}_{i1}\right), \left|\dot{q}_{i2}\right| / g_{i2}\left(\dot{q}_{i2}\right), \cdots, \left|\dot{q}_{in}\right| / g_{in}\left(\dot{q}_{in}\right)\right)$.

Example

Consider a mechanical network including four two-link revolute manipulators whose dynamics equation is modeled in [6]. Let $\mathbf{A}_1 = \begin{bmatrix}0 & 1 & 1 & 0\end{bmatrix}^T$, $\mathbf{A}_2 = \begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}^T$, $\mathbf{A}_3 = \begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}^T$, $\mathbf{A}_4 = \begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix}^T$, $\mathbf{A} = \begin{bmatrix}\mathbf{A}_1 & \mathbf{A}_2 & \mathbf{A}_3 & \mathbf{A}_4\end{bmatrix}$ and $\mathbf{B} = diag(1,1,0,0)$. For suitably selected system parameters, the results of angle tracking and angular velocity tracking are presented in Figs.1 and 2. In comparison, results obtained by using common adaptive control strategy with the same common parameters are presented in Figs.3 and 4.

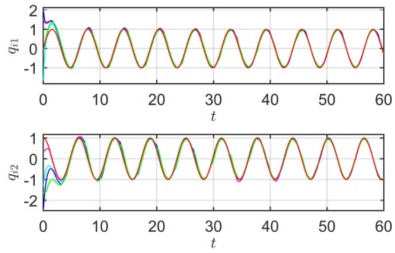

Fig.1 Angle tracking result

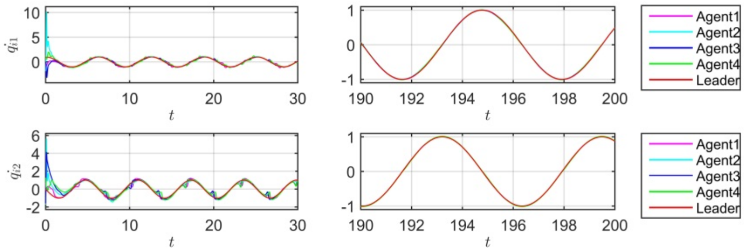

Fig.2 Angular velocity tracking result

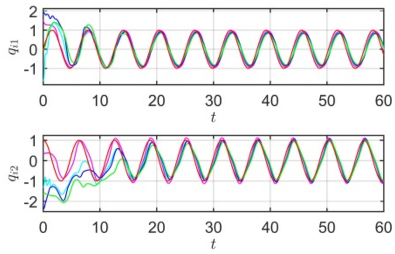

Fig.3 Angle tracking result

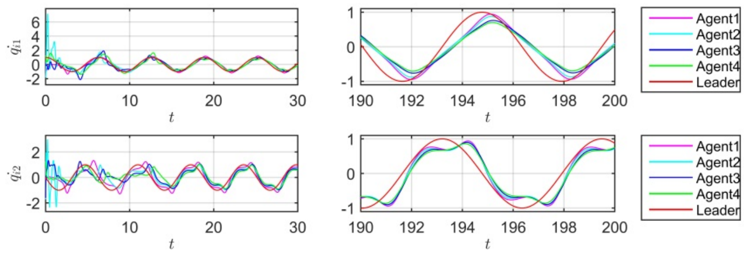

Fig.4 Angular velocity tracking result

Conclusions

We present tracking algorithm (3) for the synchronization of networked Lagrange system with uncertain parameters. The proposed tracking algorithm achieves higher control accuracy by replacing the traditional static friction model by a dynamic friction model, i.e., the Lugre friction model, which is shown to be effective in the simulation. It should be noted that the algorithm can be proved to have a weak assumption on control parameters and is therefore convenient in practical application.

References

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