Nonlinear Resonances of a Rigid-Flexible Antenna System

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Summary. This paper studied the nonlinear resonance of a rigid-flexible space antenna under three-to-one resonance condition. First, the simplified dynamic model for the antenna having two degrees of freedom is given via the assumed mode method. Then, the method of multiple scales is utilized to obtain the approximate solutions of the resonance. The numerical results show that more than one nonlinear normal mode exist over a wide range of the detuning parameter. Finally, an experimental set-up is designed to verify the approximate solutions. The results show that both the numerical and experimental results are agreement with the analytical ones.

Introduction

Large space structures usually carry one or several antenna subsystems for different space missions. During the deployment and the on-orbit service of space structures, the attitude of the antenna subsystem need to be adjusted according to different space missions. The application of manipulator to the attitude adjustment and the vibration suppression of the antenna is an important issue. In this study, the antenna system is simplified as a rigid-flexible system including a rigid arm and a flexible beam with a torsional spring at the joint and only the in-plane vibration of the antenna system is taken into consideration. Such a rigid-flexible system is usually named as the L-shaped beam in many previous researches. Recent years have witnessed numerous studies on the dynamic characteristics of the rigid-flexible multibody systems [1-4]. Researchers [5-8] have paid attention to the study on the nonlinear dynamic behavior or the vibration suppression of a flexible beam carrying an attached mass. This study deals with the analytical and experimental studies on the nonlinear resonance of a space antenna system of two degrees of freedom with the consideration of the coupling between the motion of the rigid arm and the deformation of the flexible beam.

Nonlinear resonance analysis

As shown in Fig. 1, the study focuses on the in-plane vibration of an antenna system, which consists of a rigid arm and a flexible beam. The arm is hinged to the main body of the satellite via a torsional spring and the beam is fixed at the free tip of the arm. The main body of the satellite is usually much heavier than the antenna system. Thus, the motion of the antenna system is assumed to have a little influence on the main body of the satellite. That is, the main body of the satellite can be considered as fixed in an inertial frame of reference in the study. In addition, it is assumed that the deformation of the beam is small and the rotation speed of the antenna system is slow.

An inertial frame *OXY* of reference and a body frame *oxy* of reference are established as shown in Fig. 1, where the motions of the rigid arm is described by rotation angle θ, and the deformation of the beam is represented by $w(x,y)$ in the body frame. The kinetic energy and the potential energy of the antenna system are

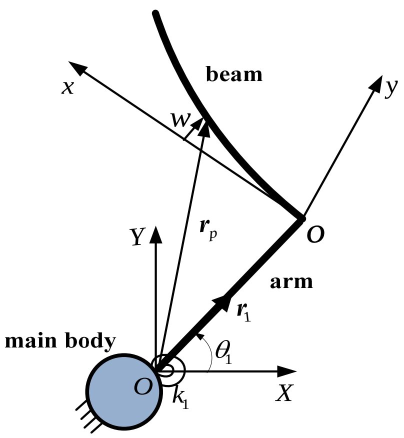

Fig. 1 The simplified model of a rigid-flexible L-shape antenna system

$$T=\frac{1}{2}m\dot{\boldsymbol{r}}_1^{\mathrm{T}}\dot{\boldsymbol{r}}_1+\frac{1}{2}J\dot{\theta}^2+\frac{1}{2}\int_0^L\rho A\dot{\boldsymbol{r}}_p^{\mathrm{T}}\dot{\boldsymbol{r}}_p\mathrm{d}x\,,\quad V=\frac{1}{2}k_1\theta^2+\frac{1}{2}\int_0^L EI\left(\frac{\partial^2 w}{\partial x^2}\right)^2\mathrm{d}x \tag{1}$$

where m, l, J and $\boldsymbol{r}_1$ are the mass, the length, the inertia moment and the position vector of the mass center of the arm, respectively. ρ, A, L and EI are the density of beam material, the cross section area of beam, the length of beam, and the bending stiffness of beam, respectively. $\boldsymbol{r}_p$ is the position vector of an arbitrary point on the beam. k_1 is the stiffness of the torsional spring. Assume that $w(x,t)=\varphi(x)q(t)$, here $\varphi(x)$ is the first mode shape of the cantilever beam. By substituting the kinetic energy and the potential energy into the Lagrange equation, the dynamic equation of the antenna system of two degrees of freedom can be derived as

$$\boldsymbol{M}(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}}+\boldsymbol{K}\boldsymbol{\eta}-\boldsymbol{P}(\boldsymbol{\eta},\dot{\boldsymbol{\eta}})=\mathbf{0} \tag{2}$$

where $\boldsymbol{\eta}=\begin{bmatrix}\theta & q/L\end{bmatrix}^T$ denotes the generalized coordinate vector. Based on the method of multiple scales, a third-order uniform approximate solution is assumed as follows

$$\boldsymbol{\eta}=\boldsymbol{\eta_0}+\varepsilon\boldsymbol{\eta}_1(T_0,T_1,T_2)+\varepsilon^2\boldsymbol{\eta}_2(T_0,T_1,T_2)+\varepsilon^3\boldsymbol{\eta}_3(T_0,T_1,T_2)+\cdots \tag{3}$$

where $T_r=\varepsilon^r t,\ r=0,1,2\cdots$, and ε is a small bookkeeping parameter. By solving the eigenvalue problem, $\omega^2\boldsymbol{p}=S_0\boldsymbol{p}$, one obtains the following conditions of 3:1 internal resonances in dimensionless parameters:

$$(k+\frac{1}{3}(db^2+1)+b^2)^2=\frac{400}{36}k\left(\frac{1}{3}(db^2+1)+b^2-0.569^2\right) \tag{4}$$

Experimental verification

The length sizes of the arm and the beam-2 were 0.11m and 0.297m, respectively. The mass of the arm and the beam-2 were 0.0749kg and 0.0185kg, respectively. The bending stiffness of beam-2 was set as 0.002Nm^2. By adjusting the clamping position, the equivalent stiffness of the torsional spring was 0.0178Nm so as to meet the requirement of $\omega_2\approx 3\omega_1$. The vibration responses of the beam-2 were measured by a laser displacement sensor, as shown in Fig. 2.

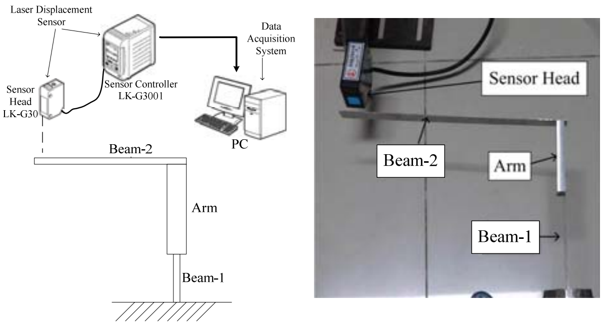

Fig. 2 Schematics and the experimental setup

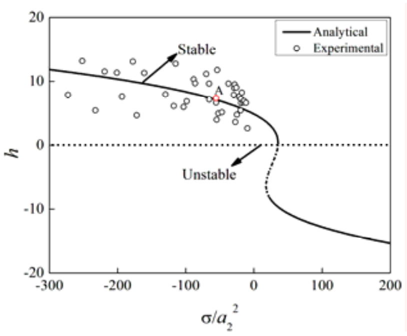

Fig. 3 Comparison of experimental and analytical results

The mode amplitude ratio for 3:1 internal resonance can be found, as shown in Fig. 3 (circle A). By changing the initial deformation of the free tip of the beam-2, dozens of experiment data are obtained. Figure 3 presents the ratio of steady-state amplitudes *h* versus σ/a_2^2. The experimental data are agreement with the analytical predictions, and the unstable solutions predicted by the analysis are not occurred in the experiment.

Conclusions

The L-shape antenna system has various internal resonances under different combinations of structural parameters. Using the method of multiple scales, one can obtain the approximate solutions of those internal resonances, including the frequency-amplitude responses and their stabilities. An important contribution of the study is to analyze the nonlinear internal resonances of the L-shape rigid-flexible structure via experimental tests. There is a good agreement between the analytical and experimental results.

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