Worst-case analysis of approximate straight-line motion mechanism with link tolerances and joint clearances

Vincent Acary[†], Narendra Akhadkar[*], Bernard Brogliato[†]
[*] *Schneider Electric, Grenoble, France*
[†]*INRIA Grenoble Rhône-Alpes, University Grenoble-Alpes, Grenoble France.*

Summary. This work deals with the approximate straight-line motion mechanism with variable link lengths and clearances at the joints, which induce unilateral constraints, impacts and friction, rendering the dynamics nonsmooth. The objective is to optimize the response of the Chebyshev lambda mechanism using a set of influencing parameters such as, link lengths and radial clearance in the revolute joints. The analysis is based on numerical simulations obtained with the projected Moreau-Jean time-stepping scheme. Finally, we propose and study a control strategy to improve the tracking of the desired trajectory that is based on a persistent contact controller.

Introduction and objectives

The industrial applications of the straight-line motion mechanisms are steam engines, oil wells, mobile robots, and the assembly line production where the requirement is the straight-line motion of the output link. In the most common case of the slider-crank mechanism, the straight line appears from the translational pair. However, it involves very precise manufacturing of the translational pair and it has very high frictional losses. The Chebyshev lambda mechanism illustrated in Figure 1 is used to transform the rotational motion into translational motion in the form of approximate straight-line with only revolute joints, thus limiting the friction losses. Usually, the performance of these mechanisms is not as desired, due to the manufacturing tolerances on the parts, clearances in the joints and the assembly tolerances, resulting in dimensional and geometrical variations [1]. The accumulated dimensional variation of the links and the kinematic joints in a mechanism assembly is given by a tolerance stack. All these imperfections tend to modify the dynamic response of the system and lead to the output deviation between the projected behaviour and the real outcome of a mechanism.
The objective in this work is to address the robustness of the system's performance with respect to the production tolerances and the clearances in the kinematic joints and to propose a control strategy to track the desired approximate straight–line trajectory. The nonsmooth dynamics approach is used to model the 2D revolute joint with clearance through a set of unilateral constraints with impact and friction, as done in [5]. First, the worst-case tolerance stack-up analysis is carried out with different link lengths and clearance values by keeping the values of coefficients of friction and restitution constant to optimize the performance of the approximate straight-line mechanism. In a second time, we evaluate a control strategy to improve the tracking of the desired trajectory.

Lagrangian formulation with bilateral and unilateral constraints

Let us consider a Lagrangian mechanical system with generalized vectors of coordinates $q \in \mathbb{R}^n$ and velocities $v \in \mathbb{R}^n$, and subjected to m_e holonomic bilateral constraints $h^\alpha = 0, \alpha \in \mathcal{E}$, and m_i unilateral constraints $g_N^\alpha \geq 0, \alpha \in \mathcal{I}$ with Coulomb's friction. The Lagrangian formalism of such a system is as follows [4],

$$\begin{cases} M(q(t))\dot{v}(t) + F(t, q(t), v(t)) = \nabla_q h(q)\lambda + G^\top(q(t))R, \quad \dot{q}(t) = v(t), \\ h^\alpha(q(t)) = 0, \quad \alpha \in \mathcal{E}, \\ \left.\begin{array}{l} U(t) = G(q(t))v \\ g_{\scriptscriptstyle N}^\alpha(q(t)) \geq 0, \quad R_{\scriptscriptstyle N}^\alpha \geq 0, \quad R_{\scriptscriptstyle N}^\alpha \, g_{\scriptscriptstyle N}^\alpha(q(t)) = 0 \\ -R_{\scriptscriptstyle T}^\alpha \in \mu^\alpha R_{\scriptscriptstyle N}^\alpha \operatorname{sgn}(U_{\scriptscriptstyle T}^\alpha) \\ U_{\scriptscriptstyle N}^\alpha(t^+) = -e_{\scriptscriptstyle N}^\alpha U_{\scriptscriptstyle N}^\alpha(t^-), \text{ if } g_{\scriptscriptstyle N}^\alpha(q(t)) = 0 \text{ and } U_{\scriptscriptstyle N}^\alpha(t^-) \leq 0, \end{array}\right\} \quad \alpha \in \mathcal{I} \end{cases} \tag{1}$$

where $M(q) \in \mathbb{R}^{n\times n}$ is the mass matrix, $F(t, q, v) \in \mathbb{R}^n$ is the vector of generalized forces. The map $G(q) \in \mathbb{R}^{2m_i \times n}$ relates the generalized velocity vector to the relative local velocity expressed in the contact local frame. The parameter $e_{\scriptscriptstyle N}$ is the coefficient of restitution, μ is the coefficient of friction. The system (1) is numerically integrated by the Siconos software [3], using projected Moreau-Jean event capturing (time-stepping) scheme [2].

Example of Chebyshev lambda mechanism

Let us consider an example of Chebyshev lambda mechanism (see Figure 1) with mass of links m_i, length of links l_i, inertia of links I_i, $1 \leq i \leq 3$. This mechanism with perfect revolute joints is described by only one generalized coordinate q=$[\theta_1]$ and each imperfect joint adds two extra degrees of freedom to the system. Thus, for three imperfect joints J_2, J_3 and J_4 with radial clearances c_2, c_3 and c_4 respectively, we select $q = [\theta_1, \theta_2, \theta_3, X_2, Y_2, X_3, Y_3]^T$. The radial clearance is defined as $c = r_1 - r_2$, where r_1 is the radius of bearing and r_2 is the radius of journal ($r_1 > r_2$). In Figure 2, O_1 and O_2 indicate the bearing and journal centers, C_1 and C_2 represent the potential contact points on the bearing and journal respectively, and the gap function is defined by $g_{\scriptscriptstyle N} = (C_1C_2)^T\mathbf{n} = c - (O_2O_1)^T\mathbf{n}$. Without closed loop control, the mechanism is actuated with a counter-clockwise torque applied at the joint 1 (J_1), $\tau = 0.2$ Nm. For this study we have selected the manufacturing tolerance of medium class (M (medium class, in our case $0.5\mathrm{mm}$ for the radial clearance)) as given in [1]. Results are compared with the case without clearance and all the link lengths are at nominal dimension(the "ideal" case). The simulations are run over time interval $T \in [0, 2]s$, with time step $1 \cdot 10^{-5}$ s, $e_N = 0.3$

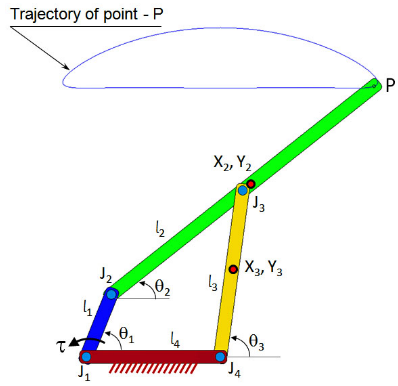


Figure 1: Chebyshev lambda mechanism.

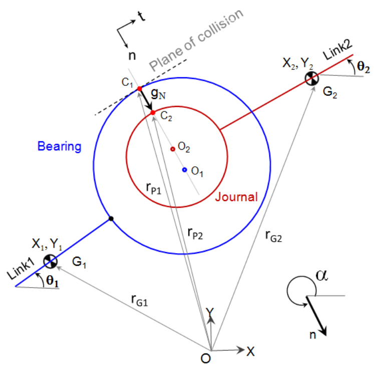
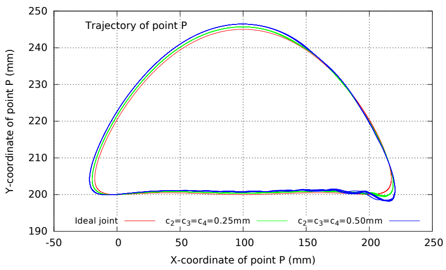
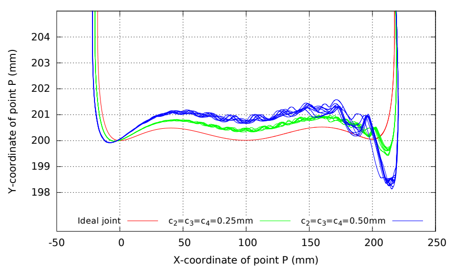


Figure 2: Planar revolute joint with clearance in a multibody system.

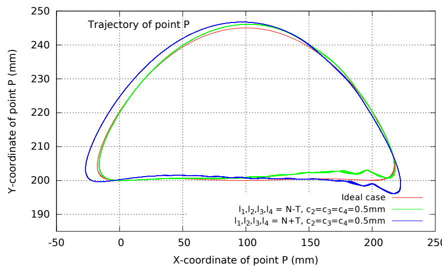
and $\mu = 0.1$.



(a) Trajectory of point P, $T \in [0.5, 2]s$.



(b) Zoomed view of approximate straight line.



(c) Trajectory of point P, $T \in [0.5, 2]s$ with link–length variations

Figure 3: Chebyshev lambda mechanism with clearance in the revolute joints J_2, J_3 and J_4.

For the input torque τ, as the clearance in the revolute joint increases the trajectory of point P starts deviating from the case with ideal revolute joints (see Figure 3(a)). In the ideal case, maximum variation of the approximate straight-line is 0.5mm (or $\pm 0.25\text{mm}$) however in case of radial clearance of 0.5mm the deviation is increased to 2mm (see Figure 3(b)). The combined effect of tolerance stack-up and the clearance in the revolute joints yields an increase up to approximately 7mm (see Figure 3(c)). It is interesting to note that the significant deviation is observed only on one side when compared to the reference trajectory.

Closed–loop control of mechanical systems with clearances

With tolerance stack-up and tolerances, we propose to improve to the system's performance by designing a control strategy dedicated to system with unilateral constraints. By controlling the torques applied in the joints J_1 and J_4 that are assumed to be perfect for a while and using a suitable feedback controller, we will show that it is possible to increase the tracking performance of the system. The controller will be designed such that it maintains closed contact in the joints with clearances (J_2 and J_3) (persistent contact controller) and then it allows to track the ideal trajectory more efficiently.

References

[1] ISO-2768: Tolerances for linear and angular dimensions without individual tolerance indications. *International Organization for Standardization*, 1989.

[2] V. Acary. Projected event-capturing time-stepping schemes for nonsmooth mechanical systems with unilateral contact and Coulomb's friction. *Computer Methods in Applied Mechanics and Engineering*, 256:224–250, 2013.

[3] V. Acary, M. Brémond, O. Huber, and F. Pérignon. Siconos. http://siconos.gforge.inria.fr.

[4] V. Acary and B. Brogliato. *Numerical Methods for Nonsmooth Dynamical Systems. Applications in Mechanics and Electronics.* Lecture Notes in Applied and Computational Mechanics 35. Berlin: Springer. xxi, 525 p. , 2008.

[5] N. Akhadkar, V. Acary, and B. Brogliato. Analysis of collocated feedback controllers for four-bar planar mechanisms with joint clearances. *Multibody System Dynamics*, 38(2):101–136, 2016.