Reference spreading trajectory tracking control: experimental analysis on a one-degree-of-freedom setup

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Summary. This work presents an experimental proof of principle for using reference spreading control to track trajectories with velocity jumps on mechanical systems with (partially) elastic impacts. A pendulum that is actuated by an electric motor at the axle and collides with a metal block when in the vertical downright position is taken as experimental platform. Trajectory tracking results show convergence of the system to the impacting reference motion even for a relatively large perturbation in initial condition. The results furthermore show that a current inadequacy in velocity estimation about the impact times limits tracking performance.

Introduction

Elastic impacts between a rigid obstacle and a mechanical system can cause sudden modifications in the relative velocity. The very small time scale of such events (compared to free motion phases) motivates the modeling assumption that the velocity variation is instantaneous. Tracking for systems with such state-triggered instantaneous velocity jumps is complicated due to the fact that the jump times of the plant and reference trajectories cannot be assumed to coincide since non-zero tracking errors, for example, will lead to differences in time of impact. Reference spreading (RS) control has been specifically developed for tracking trajectories with expected impacts or changes in contact conditions (see [1, 2, 3]) and will be analyzed experimentally in this work. More specifically, in this paper, we continue the work of [4] and present an experimental proof of principle of reference spreading control in trajectory tracking for systems with partially elastic impacts. We consider the pendulum setup in Figure 1 that, unlike in [4], has been specifically designed to shown clear restitution/bouncing and to have a relatively high effective stiffness between point of impact and point of sensing.

Reference spreading control

RS control is based on extending the considered reference trajectory before and after the expected impact events in a smooth manner and switching between the resulting ante- and post-event reference branches when an impact is detected. An example of an extended (velocity) reference trajectory is shown on the left in Figure 2. Before creating the extensions, the reference trajectory is cut into the different segments between impact events and an impact counter $j \in \mathbb{N}$ is added to each. An RS-based proportional feedback control law including feedforward is now given by

$$\tau(t,j) = \tau^d(t,j) + K(\bar{x}^d(t,j) - x(t,j)) \qquad (1)$$

where $\tau \in \mathbb{R}^m$ is the input supplied to the system, $\tau^d \in \mathbb{R}^m$ presents the feedforward, $K \in \mathbb{R}^{m \times n}$ is the feedback gain matrix, $x \in \mathbb{R}^n$ is the state of the system, and $\bar{x}^d \in \mathbb{R}^n$ represents the extended reference trajectory. Note that the control input now not only depends on time t, but also on how many impacts the tracking system has encountered, i.e. the jump counter j. See [1, 2, 3] for a more elaborate description of the control strategy.

Experimental analysis

Experimental setup

The experimental setup that is considered in analyzing the reference spreading control method is the one-degree-of-freedom impacting pendulum shown in Figure 1. A brushed-DC motor (Maxon RE35 42V 90W) is used to supply an input torque $\tau \in \mathbb{R}$ to the pendulum at the axle to make it perform a desired motion with partially elastic impacts between the impact head of the pendulum and a metal block. An Elmo VIO 5/60 current amplifier is used to drive the motor and measure the supplied current (and consequently the applied torque). The angular position $q \in \mathbb{R}_{\geq 0}$ is measured with an optical incremental encoder (HP HEDL-5540-A11, 500 CPR) at a sampling rate of 1000 Hz. The zero position corresponds to the case where the impact head is in contact with the block. We denote the estimated angular velocity by $\hat{v} \in \mathbb{R}$, which is found by applying a first order low-pass filter with cross-over frequency of 250 rad/s to the finite differencing of measured position. Position and velocity form the state of the system and are used for feedback, i.e. $x = \begin{bmatrix} q & \hat{v} \end{bmatrix}^T$. The setup is designed to show clear restitution at impact (coefficient of restitution of approximately 0.55) with negligible impact duration and deformation. To minimize the reaction forces on the bearings due to these hard impacts, the pendulum is designed such that the so-called center of percussion coincides with the impact point.

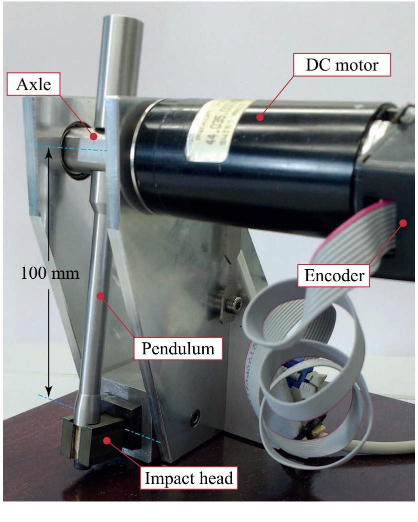

Figure 1: Experimental setup.

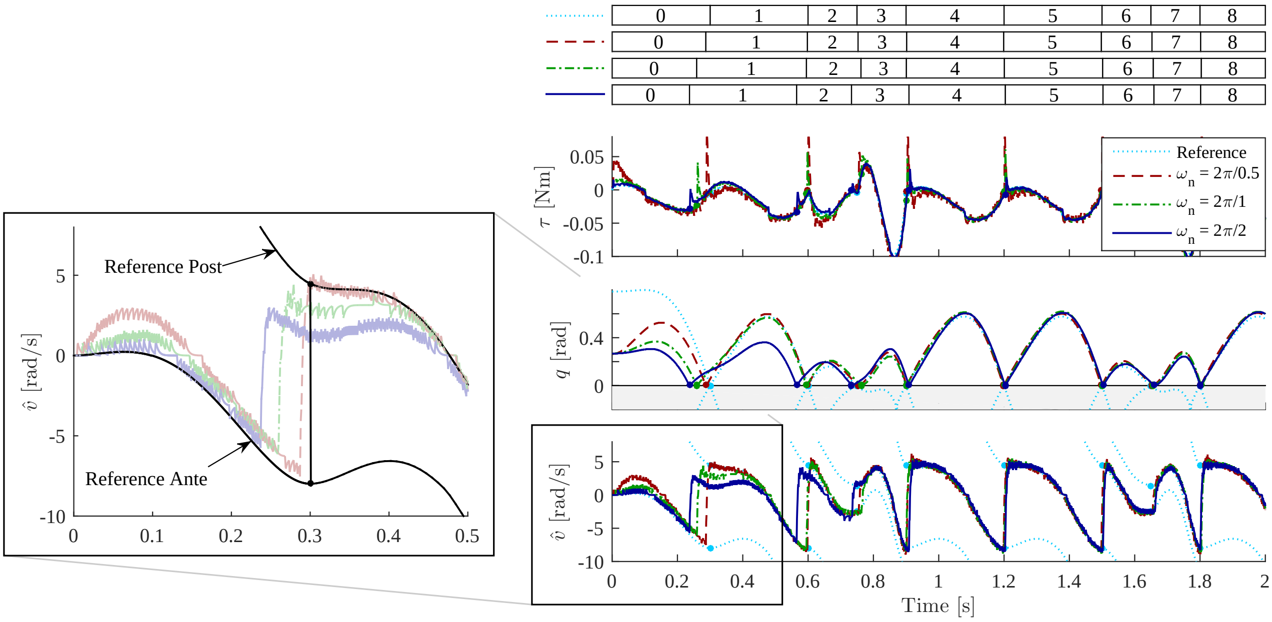

Figure 2: Trajectory tracking results for three different controller bandwidths and a zoom illustrating the extended ante- and post-event velocity reference segments. The bars on top indicate the impact times and jump counter j as a function of time.

Trajectory tracking results

Trajectory tracking experiments are performed using the reference spreading control law (1) considering a non-periodic reference trajectory that produces a melody. In this, the effect of gravity on the pendulum setup is compensated using computed torque control. The reference trajectory with extensions and the tracking results for three different control gains are depicted in Figure 2. The torques shown in the figure do not include the gravity compensation contribution. The feedback gains K for the three experiments are chosen such that the closed loop error dynamics have a damping ratio $\xi = 1$ and natural frequencies $\omega_n = 2\pi/2$ rad/s, $\omega_n = 2\pi/1$ rad/s, and $\omega_n = 2\pi/0.5$ rad/s, respectively. The impacts are detected by setting a threshold of two times the encoder resolution on position. The jump counter is incremented whenever the measured position gets below this threshold. The results present an experimental proof of principle for the reference spreading control approach. The relatively large perturbation in initial condition is rejected by the controller and the control action stabilizes the impacting reference trajectory. However, looking more closely at the input torque, it can be seen that peaks occur just after each impact. This is not caused by the control strategy, but by the incorrect estimation of velocity. The filter takes time to ‘react’ and filters out the velocity jump. This results in (incorrect) large velocity errors. The unwanted filtering effect becomes more apparent for higher bandwidth controllers as can be seen in Figure 2.

Conclusions

In this work, the reference spreading control framework has been considered and employed to perform a prescribed non-periodic motion with impacts on an actuated one-degree-of-freedom experimental setup. The setup has been specifically designed for testing the control methodology on a relatively simple system with clear restitution at impact, keeping an eye on durability with respect to these repeated collisions. Trajectory tracking results indicate that reference spreading control is a solid approach to perform prescribed motions with a system subject to (expected) external impulsive forces, i.e. hard impacts. The results furthermore show that better estimation of velocity could potentially improve tracking performance and reduce control effort. The incorporation or development of a velocity filter/estimator that can handle jumps in velocity is suggested for further research.

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