Reduced-order Modeling of Strongly Nonlinear Systems Using Measured Time Series

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Summary. We describe a new method for constructing reduced-order models (ROMs) of strongly nonlinear mechanical systems using measured transient response data. The procedure is motivated by the desire to quantify the degree of nonlinearity of a system, with the goal of updating a finite-element or other mathematical model to capture the nonlinear effects accurately. Our method relies on proper orthogonal decomposition to extract proper orthogonal mode shapes (POMs), which are inherently energy dependent, directly from the measured transient response. Using known linear properties, the system's frequency is estimated using the Rayleigh quotient, either using discrete matrices created using a finite element model or by fitting a polynomial to the discrete points and carrying out the integration analytically. An approximate frequency-energy plot (FEP) is created by plotting the estimated frequency as a function of the system's current energy. We find that the estimated FEP reveals three distinct regimes: two linear regimes and one highly nonlinear transition regime. The transition regime is characterized by large frequency changes as well as spikes that connect different modes and indicate strongly nonlinear interactions between modes. We demonstrate our method using the simulated response of a cantilever beam with a local, smooth nonlinearity attached near its free end.

Method

While the techniques to be presented are applicable to both discrete and continuous structures, it is most convenient to present them in the context of a discrete model. This is hardly a limitation in practice, considering that experimental data acquisition is inherently limited to discrete points in space, and that finite element analysis can produce a viable discrete model for most distributed systems of interest [1]. Many degrees of freedom can be accommodated, but it is often desirable to reduce the model dimension, either for computational efficiency or to avoid the need to infer quantities that are difficult to measure directly. An example of the latter is rotation about an out-of-plane axis at a point on a beam or plate undergoing bending. Such quantities can often be systematically eliminated from the equations of motion by an established technique such as Guyan reduction [2], with little effect on the behavior of the dynamical model.

Given the mass and linear stiffness properties of a system, represented by mass and stiffness matrices, we turn to the processing of time series obtained by measuring the response of the system following an input of known magnitude (typically an impulse applied to one degree of freedom). Using proper orthogonal decomposition [3], we construct a set of basis vectors which can be used to represent the response of the system over an interval when its total energy is nearly constant. Thus, the method is applicable to a damped system provided sufficient response is available at the energy of interest. Note that only the POMs (left singular vectors) are used in our method and the time series corresponding to their oscillations (right singular vectors) are not considered in this work.

Because of the nonlinearity of the system, the basis vectors obtained will, in general, depend on upon the energy in the system; they will evolve over time (albeit on a scale much slower than the system's vibration), or they will differ following inputs of different magnitudes [4]. Combining the different sets of basis vectors with the constant coefficient matrices of the linear part of the system model, we obtain energy-dependent frequency estimates and thus *energy-dependent measures of the effects of nonlinearity,* which is reflected in the measured response although it is not found in the matrices used in the post-processing.

Example

We apply our method to a cantilever beam with a local, smooth nonlinearity attached near its free end. The beam is made of steel and has a length of 1.311 m, a width of 0.045 m, and a thickness of 0.008 m. The nonlinearity is a spring attached 1.18 m from the beam's fixed end with a purely cubic force-displacement relationship and a stiffness of 5×10^{7} Nm^{-3}. We excite the beam at 0.4 m from the fixed end using a half-sine pulse with a duration of 6×10^{-4} s and varying amplitude such that the initial system energy varies from 10^{-5} to 10^{15} J. The beam's transient response is simulated at 100 points for 1 second using a finite element model. For each simulation, a set of basis vectors is extracted, and the corresponding frequency is estimated as previously described. In Figure 1, we present two sets of basis vectors and the corresponding POMs for initial system energies of (a) 10^{-3} J and (b) 10^{13} J. The estimated frequencies are plotted as functions of the total work, resulting in a frequency-energy plot. Figure 2 presents the estimated frequency-energy plot

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obtained for this system. At the low (high) energy regime the frequency-energy plot reproduces the frequencies of the beam in a fixed-free (fixed-pinned) configuration. The intermediate regime reveals the nonlinear transition between the two configurations and that the modes appear to interact with each other.

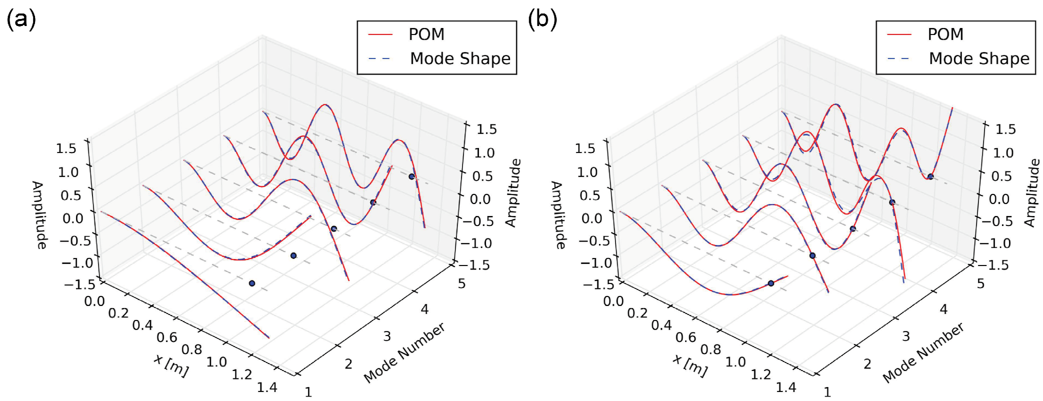

Figure 1. Comparison of basis vectors and mode shapes for initial system energies of (a) 10^{-3} J and (b) 10^{13} J. The gray dashed lines represent the beam at static equilibrium, and the blue dots indicate the nonlinearity's location.

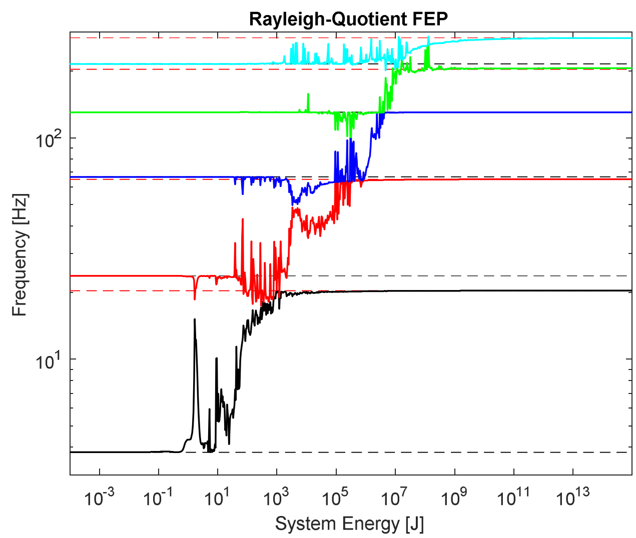

Figure 2. Estimated frequency-energy plot on a log-log scale.

References

[1] Friswell M. I., Mottershead J. E. (1995) Finite Element Model Updating in Structural Dynamics. Springer, NY.

[2] Guyan, R. J. (1965) Reduction of Stiffness and Mass Matrices. *AIAA Journal* **3:**380.

[3] Kerschen G., Golinval J-C., Vakakis A. F., Bergman L. A. (2005) The Method of Proper Orthogonal Decomposition for Dynamical Characterization and Order Reduction of Mechanical Systems: An Overview. *Nonlinear Dynamics* **41**:147-169.

[4] Kerschen G., Worden K., Vakakis A. F., Golinval J-C., (2006) Past, present and future of nonlinear system identification in structural dynamics. *Mechanical System & Signal Processing* **20:**505-592.