Two-dimensional motion of a body carrying movable internal masses

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Summary. Two-dimensional motions along a horizontal plane of a body carrying movable internal masses are analyzed. The body is in contact with the plane at three support points where dry friction forces between the body and the plane act. The friction forces obey Coulomb's law. The body carries movable masses controlled by actuators installed on the body. Motions of internal masses that do not interact with the exterior environment cause the motion of the body. Two versions of internal masses are considered: a point mass moving in a horizontal plane parallel to the support one, and a rotor with a point mass moving along a line. Both internal systems have two degrees of freedom. It is shown that the body controlled by internal masses can move from a given initial position to any prescribed terminal position in the horizontal plane. The algorithm of motion is described.

Introduction

Mobile robots are usually equipped with wheels, legs, tracks, propellers, and other external devices that interact with the exterior environment. However, it is possible to ensure the desired movement of a body in a resistive medium by means of special motions of internal masses contained inside the body. Relative motions of internal masses must be implemented by actuators installed on the body. Mobile systems containing internal masses are sometimes called vibro-robots or capsubots. This principle of motion does not require any external devices; the body can be hermetic. Such mobile robots can be useful for motions in vulnerable and hazardous media, inside tubes, and in other cases where traditional mobile systems are undesirable.

Locomotion based upon internal moving masses is possible only in the presence of resistance forces created by the exterior medium. In this paper, we restrict ourselves to dry friction forces obeying Coulomb's law.

Mobile systems whose movement is based upon motions of internal masses were considered in a number of papers, for example, in [1,2], and used for micro- and nano-positioning [3-5].

Optimal periodic motions of systems consisting of a main body and movable internal masses in the presence of external dry friction forces are considered in [6], see also [7,8]. In these papers, optimal controls are obtained that correspond to the maximum average locomotion speed of the system under constraints imposed on the relative displacement of the internal mass, its velocity or acceleration. Experimental data presented in [9,10] confirm the obtained theoretical results.

In previous papers including those mentioned above, only rectilinear progressive motions of systems with internal masses along a horizontal line were considered. In this paper, we discuss two-dimensional motions of such systems along a horizontal plane in the presence of friction forces acting between the system and the plane. Two versions of systems containing internal masses are examined; one of these versions is considered in [11]. It is shown that, under certain assumptions, the both versions can be transferred from an initial position to any prescribed terminal position in the plane. The corresponding relative motions of internal masses are proposed.

Mechanical systems

Consider a rigid body *P* of mass m_1 that can slide along a fixed horizontal plane *OXY*. Vertical axis *OZ* of the Cartesian coordinate system *OXYZ* is directed upwards. Body *P* called the main body contacts plane *OXY* at three support points $A_i,\ i = 1, 2, 3$. Since in the case of three support points the system is statically determinate, normal reactions N_i at points A_i can be found univalently.

Dry friction forces $\mathbf{F}_i$ acting between points A_i and plane *OXY* obey Coulomb's law. If point A_i slides along the plane with velocity $\mathbf{v}_i$, the friction force is defined by equations

$$\mathbf{F}_i = -fN_i\mathbf{v}_i/v_i, \quad v_i = |\mathbf{v}_i|, \quad \text{if} \quad v_i \neq 0,$$
$$|\mathbf{F}_i| \leq fN_i, \quad \text{if} \quad v_i = 0, \quad i = 1, 2, 3. \tag{1}$$

Here, f is the coefficient of friction.

We consider two versions of mechanical systems.

Version 1.

Main body *P* carries a point *Q* of mass m_2 that can move relative to the main body along a horizontal plane parallel to plane *OXY* (Fig.1). The point mass *Q* has two degrees of freedom relative to the main body and is controlled by two actuators installed on the body.

Version 2.

Two additional bodies are associated with the main body, namely, point *Q* of mass m_2 and rotor *R* of mass m_3. Rotor is a rigid body that can rotate about the vertical axis BZ' which is parallel to *OZ* and passes through point *B* of body *P*. Rotor *R* is dynamically symmetric with respect to its axis BZ'. A horizontal line directed along the unit vector **e** is connected

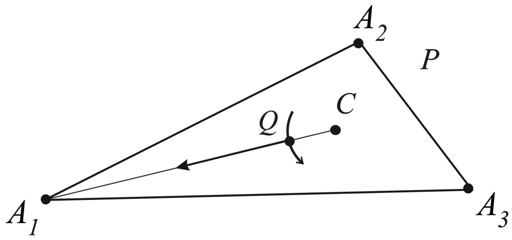

Figure 1: Two-dimensional motion (Version 1)

with the rotor and rotates with it. Point mass *Q* can move along this line; its displacement *BQ* is denoted by ξ (Fig. 2). As in Version 1, the internal bodies have two degrees of freedom relative to the main body: the angle of rotation of the rotor and displacement ξ. The relative motions of these bodies are controlled by two actuators: one of them rotates rotor *R*, and the second moves point *Q* along vector **e**.
Version 2 can be implemented in different ways; one of them is described below.

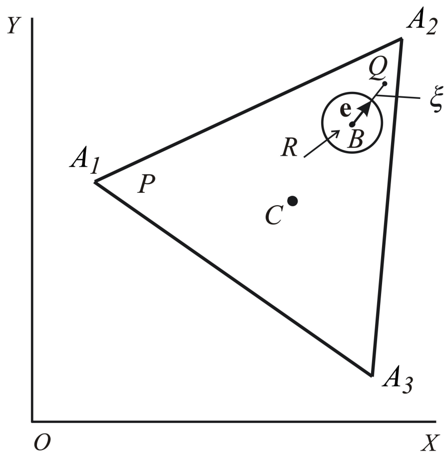

Figure 2: Two-dimensional motion (Version 2)

Version 2a.
Instead of two bodies, rotor *R* and point mass *Q*, we can consider rotor R' identical to rotor *R* and pendulum Q'. The pendulum Q' of mass m_2 and length *l* can swing about the horizontal axis perpendicular to vector **e**. Suppose that the pendulum performs small oscillations in the vicinity of one of its vertical equilibrium positions, either lower or upper one. The latter option (oscillations about upper equilibrium position) was used in the experiment [9]. In the case of small oscillations of the pendulum, the system $P + Q' + R'$ of Version 2a is equivalent to system $P + Q + R$ of Version 2; displacement $l\varphi$, where φ is the angle of deflection of the pendulum from the vertical, corresponds to the linear displacement ξ. Since Versions 2a and 2 are equivalent, we consider below only Versions 1 and 2.

Control of motion for Version 1

Let the initial and terminal positions of system $P + Q$ be given, and system is at rest at these positions. This means that the initial and terminal positions of the triangle $A_1A_2A_3$ in plane *OXY* as well as initial and terminal positions of point *Q* relative to this triangle are prescribed. The control problem is to find such motion of point *Q* relative to body *P* that transfers the system $P + Q$ from the initial position to the terminal one.
Let us describe constraints imposed on the system and the motion of point *Q*. Denote by *C* the center of mass of body *P* and assume that the vertical axis passing through point *C* is the principal central axis of inertia of the body. The projection C' of point *C* onto plane *OXY* lies within triangle $A_1A_2A_3$. Point *Q* can move arbitrarily in a horizontal plane parallel to *OXY* within a circle $|C'Q'| \leq L$, where Q' is the projection of *Q* onto *OXY*, with an acceleration $\boldsymbol{w}$ relative to body *P* bounded by the inequality

$$|\boldsymbol{w}| \leq w_0. \tag{2}$$

To design the desired motion of point *Q*, we first point out that, if point *Q* moves slowly so that its relative acceleration and velocity are small enough, then body *P* stays at rest. It follows that point *Q* can move slowly from any initial to any terminal position relative to the stationary body *P*.
Possible motion of point *Q* that solves our control problem can consist of three stages.

First, point *Q* moves slowly from its initial position to some position where the distance $C'Q'$ is equal to $l, \ l \in (0, L)$. Body *P* does not move.
At the second stage, point *Q* moves along a circle relative to body *P* so that the distance $C'Q'$ is equal to *l*. The relative velocity of this motion should be high enough so that body *P* rotates in the direction opposite to the rotation of point *Q*. To achieve the rotation of body *P*, the bound w_0 in (2) should be high enough. The motion of body *P* at this stage is not, generally speaking, a pure rotation; its center of mass *C* can also move. This stage ends, when body *P* comes to the rest, and the orientation of the triangle $A_1A_2A_3$ coincides with its terminal orientation in the plane *OXY*.
At the third stage, point *Q* should move along horizontal straight lines such that its projection Q' moves along lines $C'A_i, \ i = 1, 2, 3$. Suppose point Q' travels along line $C'A_1$; than body *P* moves progressively, and point C' moves along the same line in the horizontal plane *OXY*. In this motion, all support points move along lines parallel to $C'Q'A_1$. This motion is feasible because the normal reactions N_i at points $A_i, \ i = 2, 3$, have equal and opposite torques with respect to line $C'A_1$ and thus counterbalance each other. Using two progressive motions of body *P* along two directions $C'A_i$, it is possible to bring triangle $A_1A_2A_3$ to its terminal position. Point mass *Q* can reach its prescribed terminal position relative to body *P* by means of slow motion.

One-dimensional motion

Let us describe the one-dimensional combined motion of body *P* and point mass *Q* along one of directions parallel to $C'A_i$. Equations of this motion can be reduced to the equation

$$m\dot{v} = -fmg\mathrm{sign}v - m_2 w, \quad m = m_1 + m_2, \quad \text{if} \quad v \neq 0, \tag{3}$$

where *v* is the velocity of body *P* and w is the acceleration of point *Q* relative to *P*. For the state of rest of body *P*, equation (3) should be replaced by the inequality

$$m_2|w| \leq fmg, \quad \text{if} \quad v = 0. \tag{4}$$

Equations (3) and (4) should be supplemented by kinematic equations

$$\dot{x} = v, \quad \dot{\xi} = u, \quad \dot{u} = w, \tag{5}$$

where *x* is the displacement of body P, ξ is the displacement of point *Q* relative to *P*, and *u* is the relative velocity of point *Q*.
The boundary conditions for equations (3)-(5) are

$$x(0) = v(0) = \xi(0) = u(0) = 0, \quad x(T) = x_1, \quad \xi(T) = v(T) = u(T) = 0, \tag{6}$$

where *T* is not fixed.
The control *w(t)* must satisfy the constraints

$$|w(t)| \leq w_0, \quad 0 \leq \xi(t) \leq L$$

and boundary conditions (6) for the solution of equations (3)-(5).
As possible control satisfying all conditions imposed, the piecewise constant optimal control [6-8] can be taken that provides the maximum average speed. Under this control, the motion consists of several cycles [11]. In each cycle, point *Q* moves forward and backwards relative to body *P* with piecewise constant acceleration, whereas body *P* alternates forward motions and states of rest. The number of cycles depends on the given distance x_1 in (6).

Control of motion for Version 2

Let the initial and terminal positions of system $P + Q + R$ be given, and the system is at rest at these positions. The problem is to find such motions of rotor *R* and point *Q* relative to body *P* that transfer the system from the initial position to the terminal one.
To simplify the problem, we suppose that the triangle $A_1A_2A_3$ is equilateral, the projection C' of the center of mass *C* of the whole system $P + Q + R$ (with zero displacement $\xi = 0$ of point *Q*) lies in the center of the triangle, and the vertical axis CZ' passing through point *C* is the principal main axis of inertia of the system.
Under these assumptions, the explicit analytical solution of the control problem stated above is feasible [11]. This solution consists of three stages (Fig. 3).
At the first stage, point *Q* does not move, so that $\xi = 0$. Rotor *R* rotates about its axis. As a result, body *P* rotates about axis CZ', whereas point *C* does not move. The rotation ends at the state of rest, in which the projection B' of point *B* onto plane *OXY* lies on a line that connects the projections of the initial and terminal positions of point *C*. At the end of this stage, vector **e** should be parallel to the same line.
At the second stage, rotor *R* stays fixed relative to body *P*, while point mass *Q* moves along vector **e** that keeps its direction. As a result, body vector *P* moves progressively along the same direction. At the second stage, the center of mass *C* reaches its terminal position, and the whole system comes to rest with $\xi = 0$.

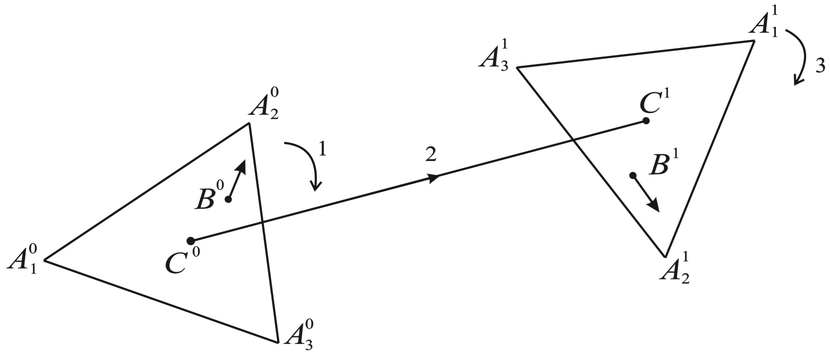

Figure 3: Stages of motion

At the third stage, as at the first one, the point mass Q stays at rest with $\xi = 0$. Due to the rotation of rotor *R*, body *P* rotates about the fixed vertical axis passing through its center of mass *C* that stays fixed. The rotation ends at the state of rest, in which the triangle $A_1A_2A_3$ comes to its terminal position. Also, in this position point *B* and vector **e** should reach their terminal states.

At all three stages, the motions are essentially one-dimensional: rotations at the first and third stages, and translation at the second one. They can be described by equations (3)-(5) and boundary conditions (6). Formulas for controls are given in [11]. In addition, certain slow motions of rotor *R* and point *Q* should be used that keep body *P* at the state of rest. Thus, the solution of the control problem stated above is obtained.

Conclusions

It is shown that the mechanical system consisting of a main body and internal movable masses attached to it and controlled by actuators, can be transferred from an initial state to any terminal state in the horizontal plane. The motion occurs in the presence of dry friction forces acting upon the main body. Two versions of internal masses associated with the main body are considered; both of them have two degrees of freedom relative to the body. The motions of internal masses that bring the system to the desired position include several stages, either rotations or translations, that are reduced to one-dimensional motions considered earlier. The results obtained can be useful for mobile robots moving in hazardous or vulnerable environment; these robots have no external devices and may be hermetic.

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References

[1] Breguet J.-M. Clavel R.(1998) Stick and slip actuators: design, control, performances and applications. Proc. International Symposium Micromechatronics and Human Science (MHS). IEEE, New York, 89-95.

[2] Fidlin F., Thomsen J.J. (2001) Predicting vibration-induced displacement for a resonant friction slider. Eur. J. of Mech. A/Solids **20**: 155-166.

[3] Schmoeckel F., Worn H. (2001) Remotely controllable mobile microrobots acting as nano positioners and intelligent tweezers in scanning electron microscopes (SEMs). Proc. International Conference Robotics and Automation, IEEE, New York, 3903-3913.

[4] Lampert P., Vakebtutu A., Lagrange B., De Lit P., Delchambre A. (2003) Design and performances of a one-degree-of-freedom guided nano-actuator. Robot Comput Integr Manuf **19**: 89-98.

[5] Vartholomeos, P., Papadopoulos E. (2006) Dynamics, design and simulation of a novel micro-robotic platform employing vibration microactuators. J. Dyn Syst Meas Control **128**: 122-133.

[6] Chernousko F.L. (2006) Analysis and optimization of the motion of a body controlled by a movable internal mass. J. of Appl Math and Mech **70**: 915-941.

[7] Chernousko F.L. (2007) Dynamics of a body controlled by internal motions. Proc. IUTAM Symposium Dynamics and Control of Nonlinear Systems with Uncertainty. Springer, Dordrecht, 227-236.

[8] Chernousko F.L. (2014) Dynamics and optimization of multibody systems in the presence of dry friction. Constructive Nonsmooth Analysis and Related Topics. Springer, New York, 71-100.

[9] Li H., Firuta K., Chernousko F.L. (2005) A pendulum-driver cart via internal force and static friction. Proc. International Conference Physics and Control. St.Petersburg, Russia, 15-17.

[10] Li H., Firuta K., Chernousko F.L. (2006) Motion generation of the Capsubot using internal force and static friction. Proc. 45th IEEE Conference Decision and Control. San Diego, USA, 6575-6580.

[11] Chernousko F.L. (2016) Two-dimensional motions of a body containing internal moving masses. Meccanica **51**: 3203-3209.