Fractional-Order Controller Design Based on the Nyquist Diagram for the Vibration Control of a Flexible Beam

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Summary. This paper proposes a fractional-order controller design method for the vibration suppression of a flexible structure. First, the process of designing a controller to meet the specifications of the Nyquist diagram is explained. An improvement to the design method that allows the attenuation level of a resonant peak to be treated as a specification is then discussed. Numerical simulations performed in this study demonstrate that the proposed fractional-order controller is more robust against spillover instability than the integer-order controller.

Introduction

In aircrafts and space structures, there are many structural components that can be considered as flexible beams. Vibrations can be harmful to the performance of such structures. In this study, fractional calculus was applied to the vibration suppression of a flexible beam, and the controller parameters were designed. The tuning of the controller parameters has been performed in some previous studies using a trial-and-error approach [1] [2]. However, this tuning process is time-consuming, and there is a risk of destabilization. As an alternative tuning method, a function that implements an optimization algorithm (e.g., the "fmincon" function in MATLAB) has been used in some studies [3] [4]. However, this complicated algorithm can place a heavy load on the computer. Therefore, in this study, a fractional-order controller was designed as an alternative non-trial-and-error approach that is simpler than previously implemented optimization algorithms.

Controller design based on the Nyquist diagram

The vibration control system shown in Fig. 1 was considered in this study. In this figure, y is the displacement of the vibratory system, f_d is the disturbance that causes the vibration, and f_c is the control force. Moreover, $G_d(s)$ is the transfer function from the disturbance to the displacement, $G_c(s)$ is the transfer function from the control force to the displacement, and $C(s)$ describes the controller. The controller function is given as

$$C(s) = Ks^q, \tag{1}$$

where K is the feedback gain, $q \in (0, 1)$ is a fractional order, and s^q is a fractional-order derivative (FOD). The loop transfer function of the control system can be described as

$$L(j\omega) = C(j\omega)G_c(j\omega). \tag{2}$$

When the vibratory system can be expressed as a one-degree-of-freedom system and the damping is small, the plot of eq. (2) in the complex plane forms a circle, as shown in Fig. 2. This is the Nyquist diagram or Nyquist circle. Point B on the circle, which is the point on the circle that is farthest from the origin of the complex plane, gives the loop transfer function value $L(j\omega_p)$ at the resonant frequency ω_p of the vibratory system. The length r of the line between point B and the origin and the angle $\theta \in (0, \pi/2)$ between this line and the imaginary axis are defined as

$$r = \left|L(j\omega_p)\right| \tag{3}$$

$$\theta - \frac{\pi}{2} = \arg\left(L(j\omega_p)\right). \tag{4}$$

From eqs. (3) and (4), the following approximations can be derived because $\arg\left(G_c(j\omega_p)\right) \approx -\pi/2$ when the damping of the vibratory system is small:

$$q = \frac{2}{\pi}\left\{\theta - \frac{\pi}{2} - \arg\left(G_c(j\omega_p)\right)\right\} \approx \frac{2\theta}{\pi} \tag{5}$$

$$K = \frac{r}{\omega_p^q\left|G_c(j\omega_p)\right|}. \tag{6}$$

Next, the peak suppression level A_p is introduced as a new specification that can be used instead of r. The closed-loop response

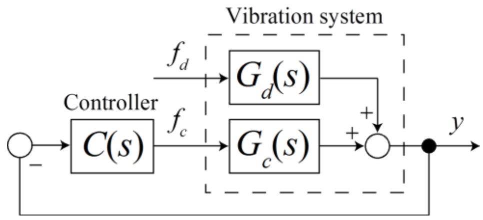


Fig. 1 Block diagram of the vibration control system

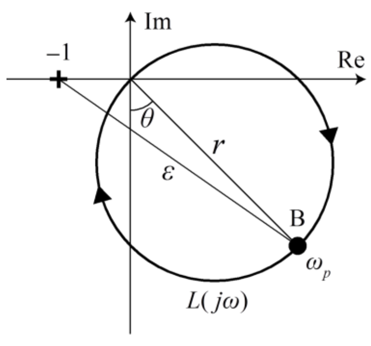


Fig. 2 Nyquist diagram

at the resonant frequency ω_p can be described as

$$\left|H(j\omega_p)\right| = \left|\frac{G_d(j\omega_p)}{1+L(j\omega_p)}\right|. \tag{7}$$

Based on this equation, the peak suppression level A_p can be given in decibel notation as

$$A_p = \left|1 + L(j\omega_p)\right|_{\mathrm{dB}}. \tag{8}$$

Moreover, ε is defined as $\varepsilon = \left|1 + L(j\omega_p)\right| = 10^{\frac{A_p}{20}}$, and the relationship between ε and r can be obtained using the law of cosines as

$$\varepsilon^2 = 1 + r^2 - 2r\cos(\theta + \tfrac{\pi}{2}). \tag{9}$$

Finally, eq. (9) yields

$$r = -\sin\theta + \sqrt{10^{A_p/10} - \cos^2\theta}. \tag{10}$$

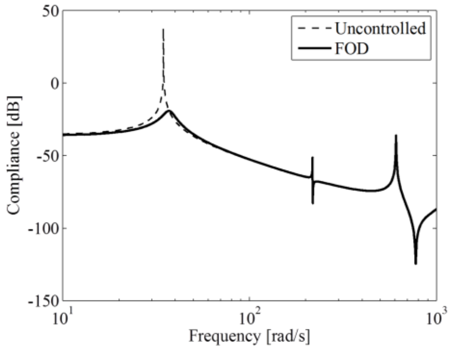
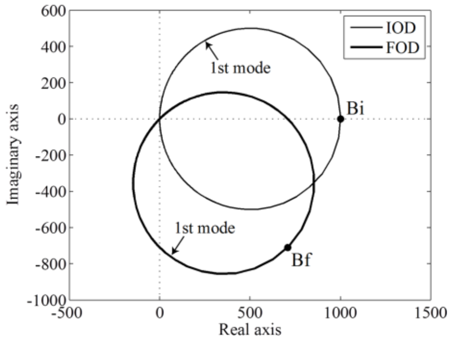
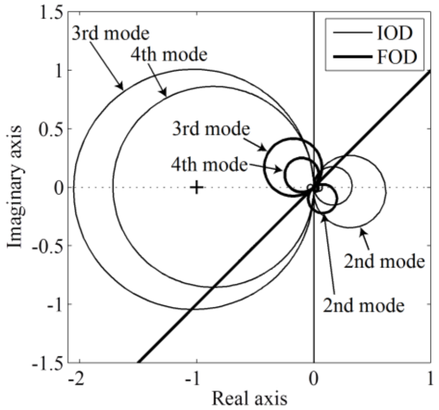


Fig. 3 Frequency response for FOD control



(a) Full Nyquist plots



(b) Magnification in the vicinity of the origin

Fig. 4 Nyquist plots for IOD and FOD control

Numerical simulations of the control of a flexible cantilever

The controller design method was then applied to the vibration control of a flexible cantilever beam. The sensor and the actuator are considered to be non-collocated. The FOD controller was compared with the integer-order derivative (IOD) controller in which $q = 1$ in eq. (1). Furthermore, the MATLAB function “ora_foc” [5] was utilized to realize s^q in the FOD controller. The transfer function of the cantilever was derived using the modal expansion method, and the vibration modes up to 10th mode were taken into account in the mathematical model of the cantilever.

In the controller design, the suppression of the first vibration mode was considered. The control specifications were set to $A_p = 60\,dB$ and $\theta = 45°$, respectively. Using the proposed method, the parameters of the FOD controller were obtained as $K = 7.35$ and $q = 0.50$, and the feedback gain of the IOD controller was obtained as $K = 1.25$.

First, the frequency responses were obtained, as shown in Fig. 3. The results demonstrate that the criterion $A_p = 60$ dB is satisfied for the FOD control. The Nyquist diagrams for the two types of control are shown in Fig. 4. As shown in Fig. 4(a), the angles of the resonant point to the imaginary axis are approximately $45°$ and 0° in the case of the FOD and IOD control, respectively. In Fig. 4(b), the Nyquist circles corresponding to some higher vibration modes for the FOD and IOD control are shown. This figure illustrates that the FOD control maintains stability, whereas the IOD control becomes unstable, as indicated by the fact that the Nyquist circles of the higher vibration modes surround the point $[-1, 0j]$. The results shown in Fig. 4(b) demonstrate that the IOD control leads to spillover instability and the FOD control avoids this instability. Therefore, from the viewpoint of robustness against spillover instability, FOD control is superior to IOD control. Spillover instability can never be ignored when considering the vibration control of flexible structures. Accordingly, FOD control is more suitable than IOD control for the vibration control of flexible structures. In the future, FOD control should be compared with robust control such as H_∞ control.

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