Investigation of the dynamics of the wiper blade around the reversal

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Summary. The behavior of windshield wiper blades around the reversal is investigated. We introduce a 2DOF analytical link model and consider dynamic and static frictions. Introducing Baumgarte's stabilization method and the slack variable method to consider strictly the effects of dynamic and static frictions, we theoretically investigate the vibrational response caused by friction and the response frequency. The theoretical predictions are in good agreement with experimental results using an actual wiper blade.

Introduction

Reducing noise generated by automobile windshield wipers around the reversals is desired. For this purpose, it is essential to clarify the behavior of the wiper blade around the reversal. In this study, we theoretically and experimentally investigate this behavior. We introduce an analytical link model with two degrees of freedom and consider two types of states; slip state in which the dynamic friction acts and stick state in which static friction acts. In the theoretical approach, the static friction is modeled by using a set-valued function. Also, the slack variable method is introduced to find the exact transition time between the slip and the stick states. Parameter studies indicate that the blade tip frequently transitions between the slip and the stick states and the frequency of the vibration caused by this transitions is close to the natural frequency of the neck of the wiper blade. The theoretical predictions are in good agreement with experimental results.

Analytical investigation and the numerical calculation results

We introduce a 2DOF analytical link model as shown in Fig. 1 [1]. The head reciprocates against a swept surface fixed horizontally. The origin of the static $x - y$ coordinate system is established as the position of the head when the blade is standing upright. As depicted in Fig 1, (u, v) and (u_3, v_3) indicate the displacements of the head and of the lip tip, respectively. θ and φ indicate the rotational angles of the neck and the lip, respectively. The initial compressive force applied to the wiper blade is expressed by pressing the support side down, where the displacement of the support side is h_d. We consider the case when the lip tip is in contact with the swept surface at all times. N and f are the normal force and the friction acting from the swept surface to the lip tip, respectively. We consider the case that the horizontal displacement of the head v is given as $v = a(1 - \cos\omega t)$. Whereas the horizontal displacement of the head is given, it is free moving in the vertical direction. Other parameters in Fig. 1 represent the characteristics of the wiper blade itself. The equations of motion of rotation of the neck, rotation of the lip and the head in the y-direction are transformed and nondimensionalized as follows:

$$\begin{aligned}&(m_\theta - m_a l_1^* \sin^2\theta)\ddot{\theta} + \Big[m_c\cos(\theta-\varphi) - m_a l_2^*\sin\theta\sin\varphi\Big]\ddot{\varphi} - m_a l_1^*\cos\theta\sin\theta\dot{\theta}^2 + \Big[m_c\sin(\theta-\varphi)\\&- m_a l_2^*\cos\varphi\sin\theta\Big]\dot{\varphi}^2 + m_a\ddot{v}^*\cos\theta - N^* l_1^*\sin\theta - f^* l_1^*\cos\theta - m_\varphi(\varphi-\theta) - c_2^*(\dot{\varphi}-\dot{\theta}) + k_1^*\theta + c_1^*\dot{\theta} = 0,\end{aligned} \tag{1}$$

$$\begin{aligned}&\Big[m_c\cos(\theta-\varphi) - m_b l_1^*\sin\theta\sin\varphi\Big]\ddot{\theta} + (m_\varphi - m_b l_2^*\sin^2\varphi)\ddot{\varphi} - \Big[m_c\sin(\theta-\varphi) + m_b l_1^*\cos\theta\sin\varphi\Big]\dot{\theta}^2\\&- m_b l_2^*\cos\varphi\sin\varphi\dot{\varphi}^2 + m_b\ddot{v}^*\cos\varphi - N^* l_2^*\sin\varphi - f^* l_2^*\cos\varphi + m_\varphi(\varphi-\theta) + c_2^*(\dot{\varphi}-\dot{\theta}) = 0,\end{aligned} \tag{2}$$

$$\begin{aligned}N^* =&(m_a\sin\theta - l_1^*\sin\theta)\ddot{\theta} + (m_b\sin\varphi - l_2^*\sin\varphi)\ddot{\varphi} + (m_a\cos\theta - l_1^*\cos\theta)\dot{\theta}^2 + (m_b\cos\varphi - l_2^*\cos\varphi)\dot{\varphi}^2\\&- k_0^*\Big[l_1^*(1-\cos\theta) + l_2^*(1-\cos\varphi) - h_d^*\Big] - c_0^*(l_1^*\sin\theta\dot{\theta} + l_2^*\sin\varphi\dot{\varphi}),\end{aligned} \tag{3}$$

where the superscript of $*$ indicates the dimensionless quantity, m_a, m_b, m_c, m_θ and m_φ are dimensionless parameters which consist of the parameters in Fig. 1, c_0^*, c_1^* and c_2^* are dimensionless viscous damping for the motion of the head, the neck and the lip, respectively. Around the reversal, the wiper blade transitions between two types of states; the slip state, for which the horizontal velocity of the lip tip $\dot{v}_3$ is nonzero, and the stick state, for which $\dot{v}_3$ is continuously zero. We model the dynamic friction $f = f_d$ acting in the slip state as follows:

$$f_d = -\mu_d(\dot{v}_3)N, \quad \mu_d(\dot{v}_3) = \mathrm{sign}(\dot{v}_3)\{A\mathrm{exp}(-B|\dot{v}_3|) + C\}, \tag{4}$$

where μ_d is the coefficient of dynamic friction, sing$(\dot{v}_3)$ is a sign function, i.e., sing$(\dot{v}_3) = 1$ for $\dot{v}_3 > 0$ and sing$(\dot{v}_3) = -1$ for $\dot{v}_3 < 0$. Next, we model the static friction $f = f_s$ acting in the stick state as follows:

$$f_s = f_{max}\mathrm{Sign}(\dot{v}_3), \quad f_{max} = (A + C)N, \tag{5}$$

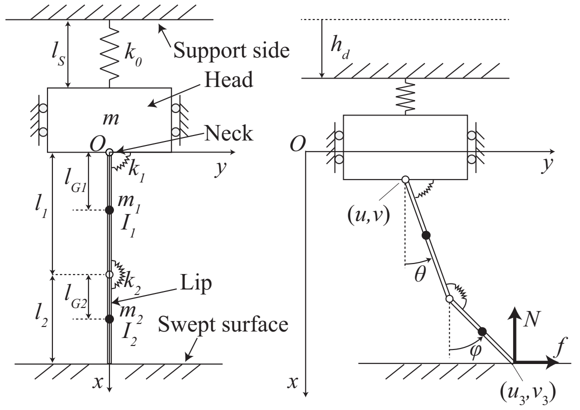

Figure 1: Analytical link model of wiper blade

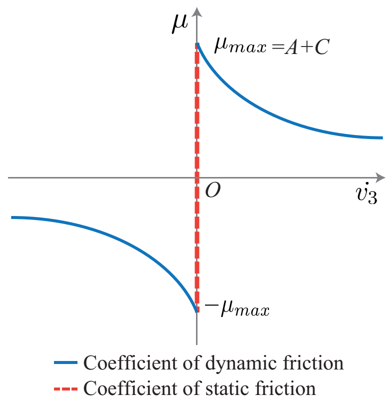

Figure 2: Coefficients of dynamic and static friction

where Sign is a set-valued function [2] whose image is a closed set [-1, 1] at $\dot{v}_3 = 0$. Figure 2 shows the relationship between the coefficient of friction and the horizontal velocity of the lip tip $\dot{v}_3$, where the coefficient of static friction μ_s is defined as $\mu_s = f_s/N$. In the stick state, the static friction balances the force from the lip tip acting on the swept surface and holds the horizontal displacement of the lip tip v_3 constant. Hence a holonomic constraint needs to satisfy $\Phi = v_3 - \alpha = 0$, where α is a constant that represents the position where the lip tip is stationary. Therefore in the stick state, we must solve the differential algebraic equation. To solve them, we introduce Baumgaete's stabilization method [3]. Around the reversal, two types of state transition occur: slip-to-stick followed by stick-to-slip. It is essential to find the exact transition time because in nonlinear systems inaccuracies in the initial condition, even if slight, result in divergent behavior. However, in general numerical calculation methods based on discrete time, the transition occurs in a discrete step size in the calculation and cannot be strictly specified. To detect the exact transition time, we introduce the slack variable method [4]. Numerical calculation results indicate that the vibration of the wiper blade contains two frequency components. The lower-frequency component is 12.9 Hz. This vibration is caused by the spring k_0 because the natural frequency of the head in the vertical direction is 8.88 Hz. The higher-frequency component is not constant and varies from 359 Hz to 692 Hz depending on the velocity of the lip tip $\dot{v}_3$. This vibration is caused by the rigidity of the neck because the natural frequency of the neck is 519 Hz. Furthermore, the lower and higher-frequency vibrations are caused by frequent transitions between the slip and the stick states.

Experimental investigation

For the experimental apparatus, an actual wiper blade was mounted on a linear actuator that swept it across a plate glass surface. Drops of water were placed on the glass plate. The behavior of the wiper blade were recorded with a high-speed video camera and were analyzed using an image analyzing software. Experimental results indicate that the vibration of the wiper blade contains two frequency components. The lower-frequency component is 78.4 Hz. It is confirmed that this vibration is caused by the spring on the head because the natural frequency of the head in the vertical direction is 20 Hz. The higher-frequency component is 9×10^2 Hz. This frequency is larger than the natural frequency of 195 Hz for the neck of the blade, where the natural frequency was obtained in a frequency-response test under conditions when the neck made no contact with the shoulder of the wiper blade. The reason for the difference between the response frequency and natural frequency is the increase in rigidity of the neck while the neck is in contact with the shoulder of the wiper blade.

Conclusions

In this present study, we introduced a 2DOF analytical link model of a wiper blade with dynamic and static frictions. The static friction is modeled by using a set-valued function. We dealt with the differential algebraic equations in the stick state using Baumgarte's stabilization method, and introduced the slack variable method for the detection of the exact transition time between the slip and the stick states. Based on these approaches, a simulation algorithm was established. The simulation indicated around the reversal that the frequent transitions between the slip and the stick states induce vibrations with high and low-frequency components. Their frequencies corresponded to the natural frequencies in the vertical motion of the head and the vibration of the neck, respectively. The experimental results using the mock-up of an actual wiper blade are in good agreement with the numerical calculation results.

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