Wave Propagation in Granular Dimers Mounted on Linear Elastic Foundation

Zaid Ahsan* and K. R. Jayaprakash**
*Department of MechSE, University of Illinois at Urbana Champaign, USA
**Mechanical Engineering, Indian Institute of Technology Gandhinagar, India

Summary. The current study dwells on the wave propagation in one-dimensional periodic granular dimer (diatomic) chain mounted on linear elastic foundation. We invoke multiple time scales and partition the dynamics of the perturbed dimer chain into slow and fast components. An analytical procedure is developed for estimating primary pulse amplitude evolution resulting in a nonlinear map relating the relative displacement amplitudes of two adjacent beads. The evolution predicted by the method of maps is in good agreement with the numerical simulation of the original system. This work forms a basis for application of the devised methodology to weakly coupled granular dimers which finds practical relevance in designing shock mitigating granular layers.

Introduction

The dynamics of granular materials over the past decade has attracted several researchers because of their unique properties [1]. The dynamics of granular systems is complex owing to the strongly nonlinear interaction between the particles governed by Hertzian theory [1, 2] in addition to the possibility of particle separation in the absence of precompression. An interesting phenomena extensively studied in a homogenous granular chain of spherical beads is the propagation of solitary waves [1, 2]. Further, the dynamics of periodic dimer chains has been quite extensively considered which revealed the existence of a countable infinity of mass ratios in $1:1$ dimer chain [3] and mass, stiffness ratios in $1:2$ dimer chains that support solitary wave propagation. The contrasting effect of effective wave attenuation has also been explored analytically, numerically and experimentally.

The practical application of granular materials in predictive designing of granular layers necessitates embedding them in elastic matrix and thus the analytical study of the effect elastic foundation on wave propagation in granular chains is necessary. In this regard, Starosvetsky et al. [4] devised a general methodology for estimating the evolution of primary pulse in homogenous granular crystals subjected to on-site perturbations. The method is based on the assumption that the primary pulse profile can be approximated by Nesterenko solitary wave [1] in the limit of weak perturbation. It was shown that the amplitude of the primary pulse on an arbitrary bead is related to that on the preceding bead resulting in a nonlinear map. Recurrent application of the map results in the prediction of amplitude evolution of the primary pulse. In contrast to the solitary/primary pulse of a homogenous chain, the light beads exhibit high frequency oscillations in a granular dimer chain during wave propagation. There exist a fast time scale governing the fast oscillations of the light beads and a slow time scale for the slowly varying localized pulse. Thus the method of maps [4] is not readily extendable to the perturbed dimers. In this study we propose a methodology based on time scale partitioning [3] and the method of maps [4] to determine the evolution of primary pulse in granular dimers mounted on damped/undamped linear elastic foundation excited by an initial impulse.

Mathematical Model and Analysis

We consider a granular dimer chain of spherical beads with no precompression mounted on damped linear elastic foundation (ref. Fig. 1). The governing equations of motion based on the Hertzian interaction is of the form

$$\begin{aligned} \ddot{x}_i &= (x_{i-1}-x_i)_+^{3/2} - (x_i - x_{i+1})_+^{3/2} - \alpha_1 x_i - \mu_1 \dot{x}_i \\ \varepsilon \ddot{x}_{i+1} &= (x_i - x_{i+1})_+^{3/2} - (x_{i+1} - x_{i+2})_+^{3/2} - \alpha_2 x_{i+1} - \mu_2 \dot{x}_{i+1} \end{aligned} \tag{1}$$

where $i = \pm 1, \pm 3, \pm 5 \ldots$, the overdots indicate derivatives with respect to the non-dimensional time (t), x_i is the non-dimensional displacement of the i^{th} bead, ε is the mass ratio between the beads, $\alpha_{1,2}, \mu_{1,2}$ are the foundation (on-site) stiffness and damping acting on the heavy and light beads respectively and the subscript $(+)$ indicates that only non-negative values need to be considered and zero otherwise. We consider $\varepsilon \le 1$, i.e. the mass of heavy bead is unity and that of the light bead is ε (ref. Fig. 1). Exploiting the time-scale separation in the response of the heavy and light beads [3], we introduce multiple time scales and study the dynamics in the limit of large normalized mass mismatch $(\varepsilon \to 0)$. The corresponding $O(1)$ system in terms of relative displacements $(\delta_{i0} = x_{i0} - x_{(i+2)0})$ is as shown in Eq. (2).

$$\ddot{\delta}_{i0} = (1/2)^{3/2}\left\{\delta_{(i-2)0}^{3/2} - 2\delta_{i0}^{3/2} + \delta_{(i+2)0}^{3/2}\right\} - \alpha_1 \delta_{i0} - \mu_1 \dot{\delta}_{i0} \tag{2a}$$

$$\delta_{(i+1)0} = \left(\delta_{i0} + \delta_{(i+2)0}\right)/2 \tag{2b}$$

Based on the assumption of weak perturbation, i.e. $\alpha_1 = \mu_1 = O(\tilde{\varepsilon})$, where $\varepsilon \ll \tilde{\varepsilon} \ll 1$ is an intermediate asymptotic parameter, the solitary wave solution of $O(\tilde{\varepsilon}^0)$ equation of Eq. (2a) is of the form $\delta_{p0} = A_p \tilde{S}_p\left(A_p^{1/4} t - j\right)$, where $p = i-2, i, i+2$ is the contact index, $j = -1,0,1$, A_p is the scaled amplitude and $\tilde{S}(t)$ is the solitary wave solution as defined in [4, 5]. Accordingly, the amplitude decay from one bead to its subsequent bead is of $O(\tilde{\varepsilon})$ i.e. $A_{i+2} = A_i + O(\tilde{\varepsilon})$ and we formulate a nonlinear map relating A_{i-2} and A_i in the form

$$f_1 A_{i-2}^{5/4} + (-f_2 + f_3) A_i^{5/4} - \alpha_1 f_4 A_i^{3/4} - \mu_1 f_5 A_i = 0 \tag{3}$$

The coefficients are defined in Table 1 [4, 5]. The amplitude evolution as predicted by the method of maps is compared with the numerical simulation of the original system (Eq. (1)) and the asymptotic model (Eq. (2)) in Fig. 2. As can be observed the amplitude evolution trend is well captured by the method of maps. Interestingly, the prediction by the method of maps even for higher values of ε are found to be fairly good [5].

Conclusions

The primary pulse amplitude evolution in granular dimers mounted on linear elastic foundation has been investigated numerically and analytically. The considered system in this work is nonintegrable and its dynamics exhibits time scale separation. Unfortunately, there are no known conventional analytical methods to analyze the dynamics of such systems. The proposed approximate methodology, which is an extension of the analytical studies [3, 4] provides a framework to study wave propagation in such complex systems. The results indicate its predictive capability in the realm of the asymptotic validity of the deduced asymptotic model. In terms of computational effort, this reduction leads to a decrease in computation time by at least one order. The method of maps provides a general trend of wave attenuation in these systems and is quite significant in such strongly or essentially nonlinear nonintegrable systems. The proposed methodology can be extended to study wave evolution in granular dimer chains with nonlinear on-site perturbations, in general nonlinear lattices that support solitary wave propagation and for weakly coupled dimer chains which has practical value in design of granular media as shock mitigation layers and material systems.

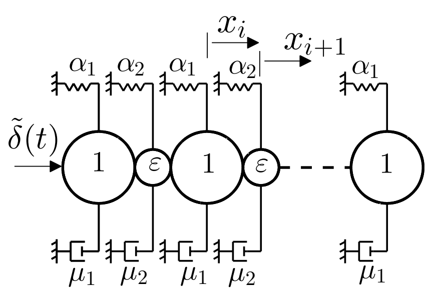

Figure 1: Schematic of granular dimer chain subject to on-site perturbations.

$$f_1 = \int_{-\infty}^{0} \tilde{S}(t+1)^{3/2} dt = 56.68$$

$$f_2 = 2 \int_{-\infty}^{0} \tilde{S}(t)^{3/2} dt = 59.8$$

$$f_3 = \int_{-\infty}^{0} \tilde{S}(t-1)^{3/2} dt = 3.12$$

$$f_4 = 2^{3/2} \int_{-\infty}^{0} \tilde{S}(t) dt = 30.66$$

$$f_5 = 2^{3/2} \int_{-\infty}^{0} \tilde{S}'(t) dt = 32.39$$

Table 1: Coefficients corresponding to Eq. (3)

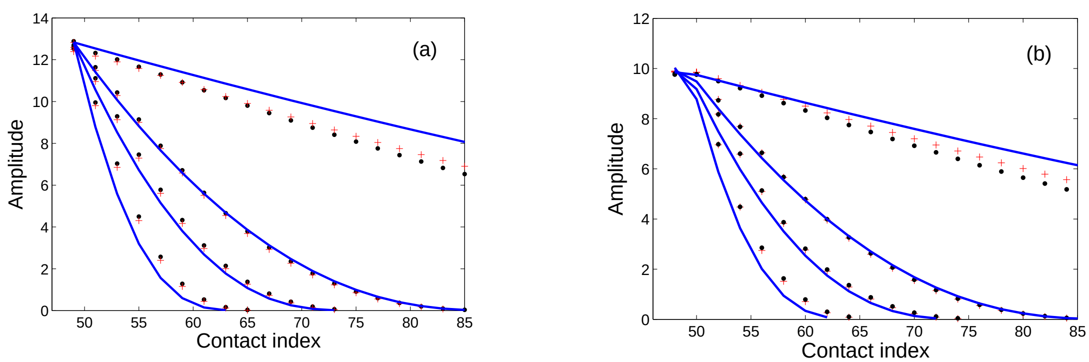

Figure 2: Primary pulse amplitude evolution in perturbed granular dimers for $\mu_1 = \mu_2 = 0, \alpha_2 = 0.01$, $\varepsilon = 0.0615$. (a) Heavy beads (b) Light beads. Curves in each panel correspond to increasing $\alpha_1 = (0.0615, 0.3, 0.5, 1)$ from top to bottom. (•) (+): Numerical simulation of original system (Eq. (1)) and asymptotic model (Eq. (2)) respectively, (−): Method of maps (Eq. (3)).

References

[1] V. F. Nesterenko (2001), Dynamics of Heteregeneous Materials. Springer-Verlag, New York.
[2] C. Coste, E. Falcon, S. Fauve (1997), Solitary Waves in a Chain of Beads Under Hertz Contact. *Physical Review E,* vol. 56(5), pp. 6104-6117.
[3] K. R. Jayaprakash, Y. Starosvetsky, A. F. Vakakis (2011), New Family of Solitary Waves in Granular Dimer Chains with No Pre-compression. *Physical Review E,* vol. 83(3), no. 036606, pp. 1-11.
[4] Y. Starosvetsky (2012), Evolution of the primary pulse in one-dimensional granular crystals subject to on-site perturbations: Analytical study *Physical Review E,* vol. 85(5), no. 051306, pp. 1-16.
[5] Z. Ahsan, K. R. Jayaprakash (2016), Evolution of Primary Pulse in Granular Dimers Mounted on Linear Elastic Foundation: An Analytical and Numerical Study, *Physical Review E,* vol. 94(4), no. 043001, pp. 1-15.