Tailoring Nonlinearity for Advanced Engineering Design: Linearization, Optimization and Practical Realization

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Summary. The intentional use of nonlinearity for engineering design, e.g., for vibration absorption and energy harvesting, has received increasing attention in recent years [1,2]. To unlock the full potential of nonlinearity, we show how its mathematical form can be tailored according to the desired dynamical performance. Three applications, namely vibration mitigation, the linearization of nonlinear resonances and the optimization of nonlinear normal modes are presented. The practical realization of nonlinear components with properties tailored through analysis is discussed.

Introduction

Our purpose in this study is to fully exploit the additional design parameter offered by nonlinear components. Specifically, we aim to tailor the mathematical form of the existing (or added) nonlinearity according to the considered design objective. Both analytical techniques relying on perturbation methods and computational methods exploiting splines and optimization are utilized for nonlinearity synthesis. The developments are demonstrated using numerical simulations and experimental measurements.

Vibration Mitigation

The first problem is the mitigation of nonlinear resonant vibrations. In view of its narrow bandwidth, when a classical linear tuned vibration absorber is coupled to a nonlinear system, it is quickly detuned when the forcing amplitude is increased (Figure 1, left). The nonlinear tuned vibration absorber (NLTVA) therefore aims to extend the range in which efficient vibration mitigation is achieved [3]. One unconventional feature of the NLTVA is that the mathematical form of its nonlinear restoring force is not imposed a priori, as it is the case for most existing absorbers. Instead, we propose to synthesize the absorber's load-deflection curve according to the nonlinearity of the primary structure. Specifically, the best performance is achieved when a *principle of similarity* is followed, i.e., the NLTVA should possess a nonlinear restoring force with the same mathematical form as that of the primary system [4]. Doing so, a nonlinear generalization of Den Hartog's equal-peak method can be devised (Figure 1, right).

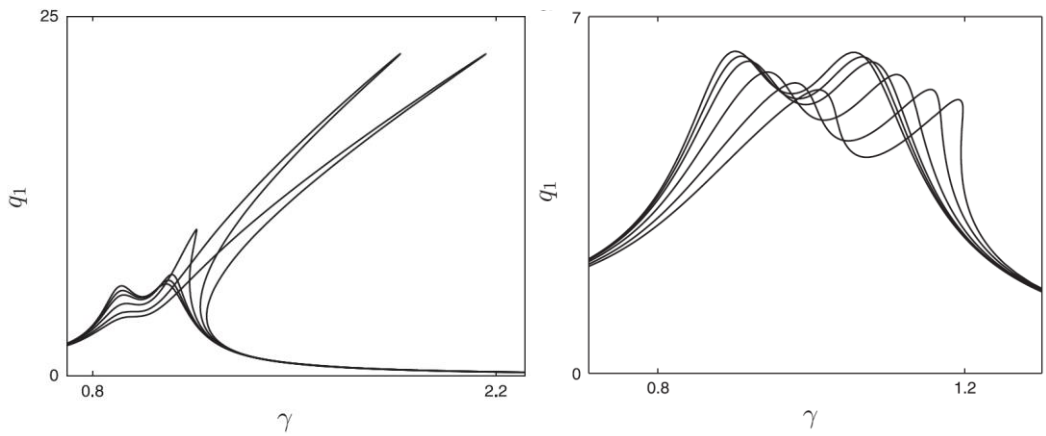

Figure 1: Vibration mitigation of a harmonically-forced Duffing oscillator, displacement vs. frequency plots. Left: linear tuned vibration absorber. Right: NLTVA.

Linearization of Nonlinear Resonances

Devices used for sensing, imaging and detection are usually required to exhibit linear behavior in their dynamic range. Since nonlinear phenomena can limit the performance of these devices, we propose a fully-passive, resonance-based approach for dealing with undesired nonlinearities in mechanical systems. Properly-tuned nonlinearities are intentionally introduced in the original system to increase the range over which a specific resonance responds linearly [5]. As for vibration mitigation, a principle of similarity is followed, i.e., the additional nonlinearity should possess the same mathematical form as that of the system. Figure 2 shows that an added cubic nonlinearity is able to enforce an almost constant resonance frequency for an initially hardening system with cubic nonlinearity.

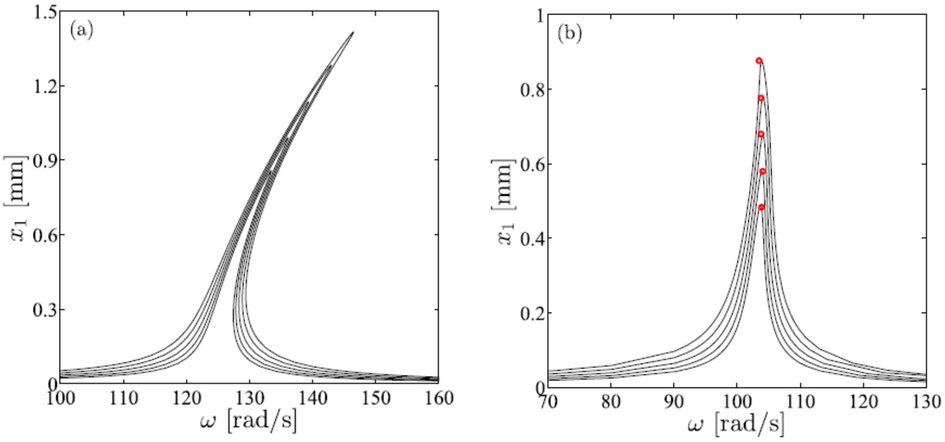

Figure 2: Linearization of a specific nonlinear resonance. Left: original system with cubic nonlinearity. Right: system linearized by addition of a cubic nonlinearity; the solid lines and red circles correspond to numerical simulations and experimental measurements, respectively.

Optimization of Nonlinear Normal Modes

We target a predefined frequency-energy dependence of nonlinear normal modes. The basic idea is to determine the nonlinearity in the system so that NNM frequencies pass through prescribed values at different energy levels Depending on the case at hand, the nonlinearity can be modeled using piecewise linear functions or splines, and a series of subproblems are solved using optimization to determine the corresponding coefficients. The nonlinearity is therefore grown from small to large energy levels. Figure 3 illustrates the procedure for two beams connected by a nonlinear joint (piecewise linear functions).

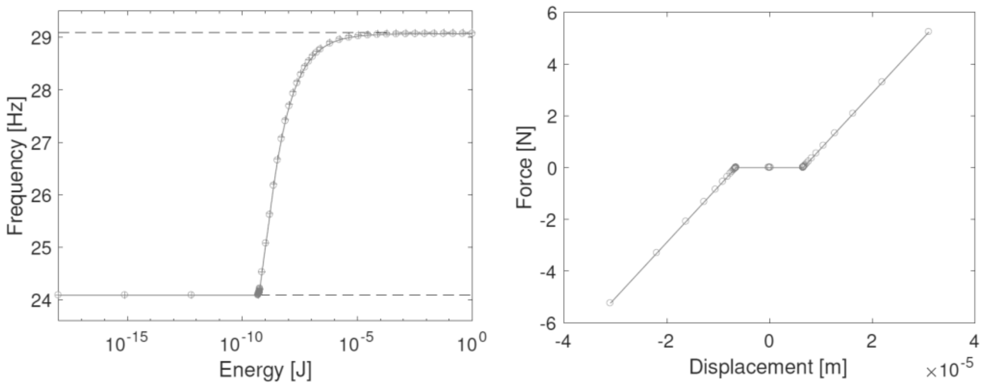

Figure 3: Optimization of nonlinear normal modes. Left: frequency-energy plot; the circles and crosses correspond to the prescribed and optimized modes, respectively. Right: nonlinearity synthesized by optimization.

Practical Realization of Nonlinear Components

Past and current efforts to construct nonlinearity in vibration absorbers and harvesters exploit cables, springs and magnets. If these elements may serve to verify the theoretical findings on academic set-ups, they lack flexibility regarding the type of load-deflection characteristics that can be achieved. For example, cables and springs with no pretension behave at first order as cubic springs. We investigate how digital piezoelectric shunts and components designed by topology optimization and built using additive manufacturing can help realize nonlinearities with prescribed mathematical forms.

References

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