Low-frequency response of controlled systems on a high-frequency parametric excitation

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Summary. The effects of high-frequency excitation on low-frequency motion were studied in many works with the help of the method of direct separation of motions. A modification of this method proposed by the author was applied earlier to a general system of mechanical oscillators with modulated high-frequency sinusoidal excitation. An essential limitation of these results is an obligatory even-numbered order of the system, which excludes from consideration mechanical or hydraulic controlled systems, which generally have an odd-numbered order. In this paper, a general set of first-order ordinary differential equations is considered. An explicit analytical expression for the modified "slow" equations is obtained. Analysis of a minimal model of a hydraulic valve is given as an example of application.

Problem definition

It is known that high-frequency excitation can lead to the modification of the behaviour of systems with respect to low-frequency motion. These effects were studied by many researchers with the help of the method of direct separation of motions proposed by I. I. Blekhman [1, 2, 3]. Some modified forms of this method with the explicit introduction of a small parameter were proposed in [4]. It was applied to a general system of mechanical oscillators with modulated high-frequency sinusoidal excitation. It was shown that five basic effects of low-frequency response on this excitation are possible. These effects are determined by the dependence of the excitation amplitude on position, velocity, and slow time. A limitation of these studies is the even-numbered order of the system. The goal of the present paper is to extend these results for a more general case of systems described with a general set of first-order ordinary differential equations with modulated sinusoidal excitation. Such a system must not consist of one-mass oscillators only, but can also include arbitrary control elements. It will allow us to consider an important class of controlled mechanical or hydraulic systems.

The systems under consideration are described with a general set of first-order ordinary differential equations with modulated sinusoidal excitation:

$$dx/dt = F(x,\tau) + \omega^{\mu}\mathrm{B}(x,\tau)\sin(\theta+\varphi) \tag{1}$$

with vectors x, F, B, φ and scalars $\mu,\ \omega,$ t, $\tau = t$, $\theta = \omega$ t, where ω>>1 and $\mu < 1$.

The objective is the equation for the averaged variable $X(\tau) = \frac{1}{2\pi}\int_0^{2\pi} x(t,\theta)d\theta$ of the form

$$dX/d\tau = F(X,\tau) + V(X,\tau) \tag{2}$$

Results

General solution

It is shown that the "slow" vibration force can be presented as $V = \omega^{2(\mu-1)}\mathrm{U}(X,\tau)$ with U=0(1). The modified method of direct separation of motions similar to that of [4] was applied to obtain an explicit general analytical expression for the components U_j of the vector function U through the components B_j , F_j and φ_j of the vector functions B, F and φ and their derivatives. This expression has the following form:

$$U_j = \frac{1}{2}\frac{\partial^2 F_j}{\partial X_k \partial X_s} B_k B_s \sigma_{ks} + \frac{\partial B_j}{\partial X_k}\sigma_{kj}\left(\frac{\partial B_k}{\partial X_s}F_s + \frac{\partial B_k}{\partial \tau}\right) + \frac{\partial B_j}{\partial X_k}\zeta_{jk}B_k\left(\frac{\partial \varphi_k}{\partial X_s}F_s + \frac{\partial \varphi_k}{\partial \tau}\right) - \frac{\partial B_j}{\partial X_s}\frac{\partial F_s}{\partial X_k}\sigma_{kj}B_k + B_j\frac{\partial \varphi_j}{\partial X_k}\cdot\Big[\left(\frac{\partial B_k}{\partial X_s}F_s + \frac{\partial B_k}{\partial \tau}\right) + \\ + \sigma_{kj}B_k\left(\frac{\partial \varphi_k}{\partial X_s}F_s + \frac{\partial \varphi_k}{\partial \tau}\right) - \frac{\partial F_k}{\partial X_s}\zeta_{sj}B_s\Big] \tag{3}$$

with $\sigma_{ks} = \frac{1}{2}\cos(\varphi_k - \varphi_s)$ and $\zeta_{ks} = \frac{1}{2}\sin(\varphi_k - \varphi_s)$. The usual convention about summing with respect to repeated indices in every term is applied.

Expression (3) enables the transformation of Eq. (1) with fast excitation to Eq. (2) for slow motion without solving the original equation. It can be used for the construction of very effective numerical algorithms in the cases when high-frequency oscillations are not of special interest, but only due to their effect on low-frequency motion.

It is worth noticing that the result takes into explicit account also the phase difference between the different components of excitation.

Example 1: One-mass oscillator (system of second order)

Consider a simple one-mass oscillator with coordinate x and velocity w

$$dx/dt = F(x,w,\tau) + \omega^{\mu}\mathrm{B}(x,w,\tau)\sin(\theta), \quad dx/dt = w \tag{4}$$

The transformation of Eq. (4) to the equations for only slow coordinate X and velocity W with the aid of the applied technique leads to the following:

$$dX/d\tau = F(X,W,\tau) + \mathrm{V}(X,W,\tau), \quad dX/d\tau = W \tag{5}$$

The general Eq. (3) for U gives in this case

$$V = \omega^{2(\mu-1)}\left(\frac{1}{4}\frac{\partial^2 F}{\partial X^2}B^2 + \frac{1}{2}\left(\frac{\partial B}{\partial W}\right)^2 F + \frac{1}{2}\frac{\partial B}{\partial W}\frac{\partial B}{\partial X}W + \frac{1}{2}\frac{\partial B}{\partial W}\frac{\partial B}{\partial \tau} - \frac{1}{2}\frac{\partial B}{\partial W}\frac{\partial F}{\partial W}B - \frac{1}{2}\frac{\partial B}{\partial X}B\right) \tag{6}$$

This result is equivalent to that of paper [4], which can be considered as a verification of the method.

Example 2: Minimal model of a valve (system of third order)

Systems of odd-numbered order are not covered by the results of paper [4], but can be considered now on the basis of the general solution (3). Consider a system of third order describing a typical hydraulic valve [5]. This system is presented schematically in Fig. 1.

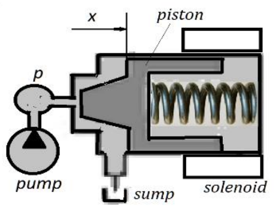

Fig. 1 Hydraulic valve

The valve is characterized by coordinate *x* and velocity w of its piston and by controlled pressure p in some hydraulic reservoir. A high-frequency excitation is provided as a magnetic force of a solenoid with the aim of avoiding a negative effect of friction on the behaviour of the valve.

The equations of this simplest controlled system consist of Eq. (4) for the motion of the piston and of one additional equation of the fluid balance for the controlled pressure:

$$dp/dt = Q \tag{7}$$

The flow rate Q depends generally on the position *x* and velocity w of the piston as well as on the pressure *p*. Also, the force in the equation of motion depends on the pressure. The transformed equations for the corresponding slow coordinate *X* and velocity W and pressure P look as follows:

$$dX/d\tau = F + \mathrm{V} + V_A,\; dX/d\tau = W,\; dP/d\tau = Q + Q_A \tag{5}$$

The additional terms V_A and Q_A in comparison to the case of a one-mass oscillator (Eq. (5)) are

$$V_A = \omega^{2(\mu-1)}\frac{1}{2}\frac{\partial B}{\partial p}\left(\frac{\partial B}{\partial W}Q - \frac{\partial Q}{\partial W}B\right),\; Q_A = \omega^{2(\mu-1)}\frac{1}{4}\frac{\partial^2 Q}{\partial W^2}B^2 \tag{6}$$

Thus, the vibrational force extra to that of the one-mass oscillator occurs only in the case that the high-frequency excitation *B* depends on pressure. In addition, either the flow rate Q or the excitation amplitude B must depend on velocity *w*. A non-trivial additional vibrational flow rate Q_A takes place only in the case of a non-linear dependence of flow rate Q on velocity.

It should be noticed that the most typical case in praxis is a linear dependence of Q on velocity due to the pump effect of the piston [5], which can be presented as $\frac{\partial Q}{\partial W} = -A$. Here, A denotes the effective area of the piston. If B in this case depends only on pressure, the expressions (6) take an especially simple form:

$$V_A = -\omega^{2(\mu-1)}\frac{1}{4}\frac{\partial B^2}{\partial p}A,\; Q_A = 0 \tag{7}$$

Typically, the excitation amplitude B increases with pressure p. This means that the additional vibration force is always negative and can lead to bigger values of the controlled pressure than specified. This effect should be taken into account in praxis.

Acknowledgements

This study was supported by the Russian Ministry of Education and Science

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