Uncertainty Quantification and Response Reliability for a Nonlinear Resonant MEMS T-beam Structure Undergoing 1:2 Autoparametric Resonance

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Summary. This work is focused on Uncertainty Quantification (UQ) and reliability analysis for a nonlinear resonant MEMS T-beam structure undergoing 1:2 autoparametric internal resonance. Linear analysis of the T-beam structure gives its modal properties. The nonlinear Lagrangian and the two lowest modes of interest then provide a two-mode nonlinear model. The nonlinear response is significantly dependent on material properties and dimensions, and thus on uncertainties in these parameters. Sensitivity analysis of the linear elastic structure allows for reduction in the number of parameters affecting the modal properties. Generalized Polynomial Chaos (gPC) technique is used to generate Response Surfaces of multi-dimensional uncertainty. The Quantity of Interest is the response of the first mode, and its variation in the presence of dimensional uncertainties allows one to evaluate variations in the response of a large set of fabricated devices, and hence the reliability of the nonlinear structure.

Introduction

UQ and its techniques deal with the quantification of effects of uncertainties in inputs and model parameters on the uncertainty in outputs of a given system. UQ in MEMS has received much attention in recent years due to significant variations in fabrication, material properties, and even the analytical models that are applicable in different operational regimes of the physics covered during operation. While MEMS devices provide advantage of size and speed, the geometric scaling presents some unique issues to be addressed before successful manufacture and application of reliable MEMS devices can become prevalent. The fabrication of these devices, which is built on the methods developed for chip manufacture, has to be improved substantially in order to provide accurate outcomes in design as well as geometric characterization of devices. These two issues essentially convert into the formal definitions of Aleatoric and Epistemic uncertainties in MEMS devices [1]. Quantification of these uncertainties can allow one to understand the limitations in the performance of a set of nominally identical devices and their operations within specified limits or bounds.

A large number of UQ techniques have been introduced in the literature. The most widely used method is the Monte Carlo (MC) technique. In this computationally intensive method, the deterministic system model is run thousands of times with samples of input parameters and the outputs are calculated at each run. Then, given the nature of distributions of input parameters, distributions of output variables can be approximated. Other techniques involve modification of the model or alternative output inference methods. Reliability Methods are used to optimize design based on targeted output. Sensitivity Analysis is done to bring out the criticality of parameters in the model. Response Surfaces are used as surrogates to simplify the model so that simulations can be run on them in a computationally feasible manner. Allen et al. [2] described the limitations of direct simulations in uncertainty analysis compared to a reliability based method. Agarwal and Aluru [3] did a stochastic analysis of an electrostatically actuated MEMS using gPC approach and quantified the outputs (e.g., the structural deformation).

Uncertainties in MEMS which utilize nonlinear phenomena for operation have not been studied. It is well-known that nonlinear resonators are highly sensitive to parameters of the system. When a large number of parameters are present, finding critical parameters and quantifying their effect on the final output is a challenging process. Complexity of governing models also makes direct repeated MC simulations practically a non-viable option.

In this work, a T-beam nonlinear microresonator based on 1:2 internal resonance [4] is studied for the effect of uncertainties in system parameters on its operation and performance. The focus is on techniques that can be used to effectively understand the effects of uncertainties in model parameters on the output characteristics of the device.

Model of T-beam Resonator and Nonlinear Dynamics

The T-Beam resonator [4] utilizes nonlinear energy transfer between the directly excited second mode and the autoparametrically excited first mode. Figure 1 shows a schematic of the device. It consists of a three-beam structure. A comprehensive nonlinear model of this structure has been developed by Vyas et al. [4]. Planar flexural vibrations of this structure (in the plane of the paper) are modeled based on Euler-Bernoulli beam theory. Bottom beams can be actuated, either by electrostatic actuation [4] or piezoelectric actuation (used here). A Lagrangian approach was used by Vyas et al. to develop the nonlinear model. The model included quadratic nonlinearities due to coriolis acceleration, cubic nonlinearities due to mid-plane stretching and curvature of beam, and effects of residual stresses. The device was designed such that it undergoes 1:2 autoparametric resonance. Thus, the two lowest modal frequencies of the overall structure are close to a ratio of 1:2. When the bottom beams are actuated near the 2nd resonant frequency, energy transfer takes place internally and the first mode is activated at 1st resonant frequency. Achieving the desired frequency ratio is dependent on all the geometric and material parameters of the system.

The equations describing the nonlinear transverse vibrations of the system can be developed using a two-mode reduced-order modeling approach. The linear mode shapes and frequencies of the lowest two modes are obtained by

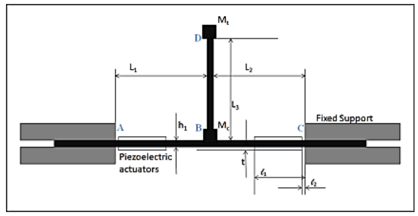

Figure 1: Schematic of the T-beam Resonator.

applying Hamilton's principle to the Lagrangian. This gives 3 equations of motion for 3 beams with 12 boundary conditions:

$$\ddot{v}_i + [N_i J_i(v_i e_{T1} + v_i e_{T2}) / r_i \nu_i^3 k_i^2](\partial^2 v_i / \partial s_i^2) + \alpha_i(\partial^4 v_i / \partial s_i^4) / r_i \nu_i^4 = 0,\ i = 1,2$$

$$\ddot{v}_3 + \alpha_3(\partial^4 v_3 / \partial s_3^4) / r_3 \nu_3^4 = 0 \qquad + 12\ \textit{boundary conditions}$$

The response of the nonlinear 2-mode model is obtained by the slow-flow equations [4]:

$$a_1' = -\zeta_1 a_1 - \Lambda_1 \omega_1 a_1 a_2 \sin(2\beta_1 - \beta_2), \quad a_1\beta_1' = (\sigma_1 + \sigma_2) a_1 / 2 - \Lambda_1 \omega_1 a_1 a_2 \cos(2\beta_1 - \beta_2),$$

$$a_2' = -\zeta_2 a_2 + \Lambda_2 \omega_2 a_1^2 \sin(2\beta_1 - \beta_2) + \Lambda_3 F \sin\beta_2 / \omega_2, \quad a_2\beta_2' = \sigma_2 a_2 - \Lambda_2 \omega_2 a_1^2 \cos(2\beta_1 - \beta_2) + \Lambda_3 F \cos\beta_2 / \omega_2$$

Here a_1 and a_2 are the amplitudes of the first mode and the second mode, respectively. The response of the nonlinear system depends on the internal mistuning σ_1 and the coefficients Λ_1, Λ_2, and Λ_3. These in turn depend on modal properties of the linear model which depend on the geometric and material parameters, prone to fabrication error.

A typical response of the nonlinear system is shown in Figure 2. It shows response of the nominal system (exact internal resonance, σ_1=0), and one with significant deviations from internal resonance (σ_1=2).

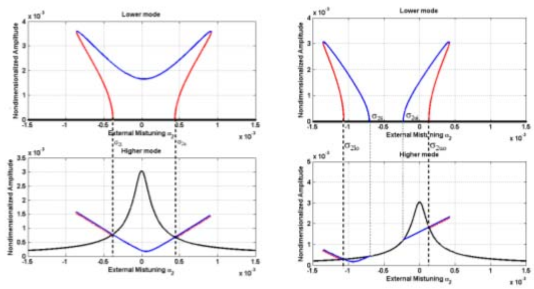

Figure 2: Response amplitudes of first and second modes; nominal system and system far from exact resonance.

Uncertainty Quantification

The focus of uncertainty analysis is on understanding the effect of manufacturing errors, and ultimately the number of devices that will pass some specified performance criteria. Here we chose the criterion that a device is acceptable if the response of the first mode at exact external resonance (σ_2=0) is at least 10% of the response for the nominal design. The steps followed for this analysis consist of the following: 1) a sensitivity analysis of the linear structure – reduces the number of parameters from 15 to 5; 2) a gPC analysis of the linear system – gives a response function relating the parameters in slow-flow equations to the 5 physical parameters; and 3) a MC simulation of the slow-flow system to evaluate the performance. The ultimate outcome (number of acceptable devices) is shown in Figure 3.

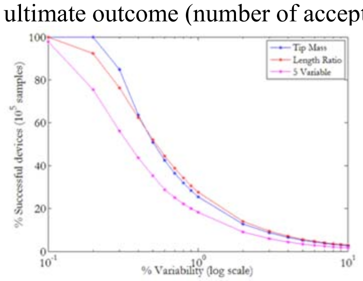

Figure 3: Number of successful devices as a function of the variation in parameters from the nominal values.

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