# Non-smooth modelling of a periodic structure with contact-friction and aero-elastic couplings

Miroslav Byrtus*, Michal Hajzman* and Ladislav Pust **
*Department of Mechanics, University of West Bohemia, Pilsen, Czech Republic*
***Institute of Thermomechanics AS CR, v.v.i., Prague, Czech Republic*

*Summary*. The paper presents a generalized model of a periodic structure (bladed disk) which contains contact-friction and aero-elastic couplings. Such structures are usually employed in e.g. steam turbine bladed disk to suppress blade vibration and over-tune the dynamic blade characteristics. A complex non-smooth model is presented in a general form which can be further used for analytical-numerical investigation of different dynamical phenomena like sliding and grazing bifurcation.

## Introduction

The aero-elasticity issue stays actual because modern aerofoils and turbine blades are designed for higher efficiencies and higher power under higher operational temperatures and flow rates. Higher operational safety and economical demands force the designers to be more precise during phase of design with respect to operational condition laying out of the area with loss of stability [1, 2, 3]. The fluid-induced forces create an aero-elastic couplings between the aerofoils and the fluid flow. Moreover, in the case of periodical structures (gas or steam turbine blades in bladed disks) the aero-elastic coupling influences not only the single blade but the adjacent blades as well.

The blade dynamics can become quite interesting especially in cases when the adjacent blades are mutually interconnected by any kind of shrouding. In the case of blade vibration, bending and torsional modes are the most important and moreover the bending and torsional motion are coupled because of non-symmetrical blade profiles [4]. There are many experimental works investigating experimentally the conditions of instability origin, e.g. [5, 6]. The paper deals with the modelling and dynamical analysis of a periodic blade system with aero-elastic and contact-friction couplings. The aim is to provide mathematical formulation of such a system in terms of non-smooth dynamics. The aim is to model and investigate the influence of aero-elastic effects, which can from certain boundaries pump the energy into the system, along with the contact-friction couplings , which should serve as energy sink. Since the contact-friction couplings have non-smooth character, different dynamical scenarios can occur during the system running (e.g. sliding and grazing bifurcations).

## Bladed disk including contact-friction and aero-elastic couplings

Further, it is assumed the bladed disk has $N_B$ blades which are created by identical airfoil profiles. Each blade is modelled by the approach presented in the previous section, i.e. it comprises two degrees of freedom (bending and torsion) and moreover these two motion are mutually coupled by so called bending-torsion coupling, see [4]. Usually, in steam turbine applications, the bladed disks are equipped with different kinds of shrouding, which causes that the system of blades mounted on a rotating disk become more stiff, especially with respect to axial flow direction.

In Fig. 1, the blade cascade of a bladed disk is depicted in a plane view. The axis of rotational symmetry designates the axis of rotor symmetry which is the bladed disk attached to. Further, it is assumed that the flow direction is parallel with the blade chords. The shrouding is supposed to be mounted at tips of the blades and it is modelled by means of two lumped springs representing bending $k_{shb}$ and torsional $k_{sht}$ stiffness of each shrouding section between two adjacent blades.

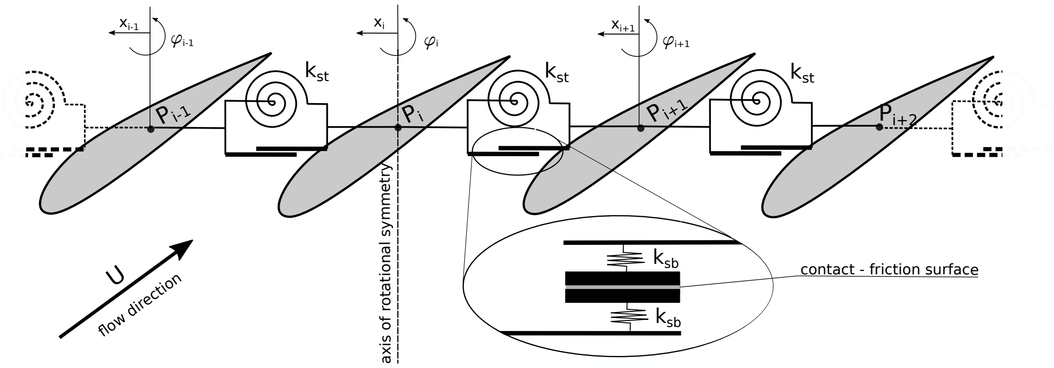


Figure 1: Bladed cascade section with contact-friction shrouding coupling modelling.

The detailed derivation of the linearized mathematical model of a disk with aero-elastic couplings is described in [7]. Here, the model is completed by the influence of contact-friction shrouding coupling. It can be advantageously written in matrix form

$$\mathbf{M}_{BD}\ddot{\mathbf{q}}_{BD} + \mathbf{C}_{BD}\dot{\mathbf{q}}_{BD} + \mathbf{K}_{BD}\mathbf{q}_{BD} = \mathbf{f}^{E}_{BD} + \mathbf{f}^{CC}_{BD} + \mathbf{f}^{FC}_{BD}, \tag{1}$$

where $\mathbf{M}_{BD}$, $\mathbf{C}_{BD}$ and $\mathbf{K}_{BD}$ are rectangular of order $2N_B$ mass, damping and stiffness matrices of a complex bladed disk model. Right hand side of (1) contains force vectors of contact coupling $\mathbf{f}_{BD}^{CC}$ and friction coupling $\mathbf{f}_{BD}^{FC}$. Vector of generalized coordinates is of following form $\mathbf{q}_{BD} = [\ldots, x_i, \varphi_i, \ldots]^T \in \mathbb{R}^{2N_B}$, where index $i = 1, \ldots, N_B$ designates the particular blade.

**Local single blade non-smooth model formulation**

The mathematical model contains both piecewise-smooth and discontinuous functions. Therefore, the model can be advantageously written in a form of autonomous, in general discontinuous, system of ODEs with local discontinuity boundaries [8]. Before, a generalized state-space has to be defined in following way $\mathbf{x} = [\ldots, x_i, \varphi_i, \tau_i, \ldots, \dot{x}_i, \dot{\varphi}_i, \dot{\tau}_i, \ldots]^T$, where $\tau_i$ represents a nondimensional time of each blade, which is connected to forcing frequency. First, let us deal with the discontinuity boundaries description. Let us focus on a particular, let say $i$-th contact-friction coupling. In general, we have two boundaries. The first

$$\Sigma_{i,i+1}^C := \{\mathbf{x} \in \mathbb{R}^{6N_B} : H_{i,i+1}^C(\mathbf{x}) := \varphi_i + \varphi_{i+1} = 0\}, \quad S_{i,i+1}^C := \{\mathbf{x} \in \mathbb{R}^{6N_B} : H_{i,i+1}^C(\mathbf{x}) > 0\}. \tag{2}$$

$\Sigma_{i,i+1}^C$ represents switching boundary between normal contact and release phases in the shrouding. The subspace $S_{i,i+1}^F$ contains all state-space points, where the normal contact between blades $i$ and $i+1$ occurs. The friction switching boundary can be then expressed as follows

$$\begin{gathered}\Sigma_{i,i+1}^F := \{\mathbf{x} \in S_{i,i+1}^C : H_{i,i+1}^F(\mathbf{x}) := \dot{x}_i - \dot{x}_{i+1} = 0\}, \quad S_{i,i+1}^{F+} := \{\mathbf{x} \in S_{i,i+1}^C : H_{i,i+1}^F(\mathbf{x}) > 0\}, \\ S_{i,i+1}^{F-} := \{\mathbf{x} \in S_{i,i+1}^C : H_{i,i+1}^F(\mathbf{x}) < 0\}\end{gathered} \tag{3}$$

and it forms a half-subspace in the global state space of the system. $S_{i,i+1}^{F+}$ nd $S_{i,i+1}^{F-}$ define two disjoint subspaces regarding the friction coupling between two adjacent blades. Then, the degree of smoothness of the system in $S_{i,i+1}^C$ is 1, that is we consider a Filippov system [8].

The final dynamical model of a bladed disk with contact-friction couplings can be formulated by means of the unions of local non-smooth models of every particular blade within the bladed disk. For the $i$-th blade holds

$$\dot{\mathbf{x}}_i = \begin{cases} \mathbf{0}, \quad \text{if} \quad \mathbf{x}_i \notin S_{i-1,i}^C \wedge \mathbf{x}_i \notin S_{i,i+1}^C \\ \mathbf{F}_{i-1,i}^C(\mathbf{x}) f(S_{i-1,i}^C, \mathbf{x}) + \mathbf{F}_{i,i+1}^C(\mathbf{x}) f(S_{i,i+1}^C, \mathbf{x}) + \mathbf{F}_{i-1,i}^{F+}(\mathbf{x}) f(S_{i-1,i}^{F+}, \mathbf{x}) + \mathbf{F}_{i-1,i}^{F-}(\mathbf{x}) f(S_{i-1,i}^{F-}, \mathbf{x}) + \\ \quad + \mathbf{F}_{i,i+1}^{F+}(\mathbf{x}) f(S_{i,i+1}^{F+}, \mathbf{x}) + \mathbf{F}_{i,i+1}^{F-}(\mathbf{x}) f(S_{i,i+1}^{F-}, \mathbf{x}) \end{cases}, \tag{4}$$

where $f(\mathcal{A}, x)$ is a binar function returning 1 if the element $x$ belongs to $\mathcal{A}$, 0 otherwise. The mathematical model (4) can be further completed by sliding vector fields, which define the behaviour of the system at switching boundaries. Right hand side functions in (4) depend on the structure matrices given in (1).

## Conclusions

The paper presents the way how to model the aerodynamic stability of a bladed disk which is created by blade-airfoil profiles mutually interconnected by shrouding at their tips. The aero-elastic coupling model is based on Theodorsen nonstationary fluid flow model. The shrouding is considered by means of contact-friction couplings. Based on the model formulation, the dynamics can be studied in terms of non-smooth system behaviour. Especially, the aim is to analyse mutual dynamical interaction of aero-elastic couplings of blade profiles with shrouding dissipation effects.

To perform all the analyses numerically, an in-house software in MATLAB code has been developed and used to analyse the bladed disk. This tool is supposed to use for different numerical analyses of the structures regarding the non-smooth dynamic phenomena.

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## References

[1] Fung Y.C. (1993) An Introduction to the theory of aeroelasticity, Dover Publications, Inc.

[2] Hodges D.H., Pierce G.A. (2002) Introduction to structural dynamics and aeroelasticity, Cambridge University Press.

[3] Tondl A., Ruijgrok T., Verhulst F., Nebergoj R. (2000) Autoparametric Resonance in Mechanical Systems, Cambridge University Press.

[4] Hayat K. et al. (2016) Flutter performance of bend-twist coupled large-scale wind turbine blades, *Journal of Sound and Vibration* **370**:149-162.

[5] Ertveldt J., Lataire J., Pintelon R., Vanlanduit S. (2012) *Flutter speed prediction based on frequency-domain identification of a time-varying system*, Proceedings of ISMA2012-USD2012, 3013-3024.

[6] Hobeck J.D., Inman D.J. (2016) *Dual cantilever flutter: Experimentally validated lumped parameter modeling and numerical characterization*, Journal of Fluid and Structures **61**:324-338.

[7] Byrtus M., Hajzman M., Dupal J., Polach P. (2016) *Dynamic phenomena of a blade system with aero-elastic coupling*, Proceedings of ISMA2016-USD2016, 3013-3024.

[8] di Bernardo M., Budd C.J., Champneys A.R., Kowalczyk P. (2008) Picewise-smooth dynamical systems (Theory and Applications), Springer-Verlag London Limited.